

Review exercises 2 Differential calculus, integral calculus

Problems

R2.1 Decide whether the statements below are true or false:

- a) "The derivative (derived function) of a function is a function."
- b) "The derivative (rate of change) of a function at a particular position is a number."
- c) "The function f has a local maximum at $x = x_1$ if $f'(x_1) = 0$ and $f''(x_1) > 0$."
- d) "If $f''(x_2) = 0$ and $f'''(x_2) < 0$, then the function f has a point of inflection at $x = x_2$."
- e) "If $g' = f$, then g is an antiderivative of f ."
- f) "f with $f(x) = 2x + 20$ is an antiderivative of g with $g(x) = x^2$."
- g) "f with $f(x) = 3x$ has infinitely many antiderivatives."
- h) "The indefinite integral of a function is a set of functions."

R2.2 Determine the value $f(x_0)$, the first derivative $f'(x_0)$, and the second derivative $f''(x_0)$ of the function f at the position x_0 :

- a) $f(x) = 4x^2(x^2 - 1)$ $x_0 = -1$
- b) $f(x) = (-3x^2 + 2x - 1) \cdot e^x$ $x_0 = -2$
- c) $f(x) = (x^2 + 2) \cdot e^{-3x}$ $x_0 = -\frac{1}{3}$

R2.3 For the given cost function $C(x)$ and revenue function $R(x)$ determine ...

- i) ... the marginal cost function $C'(x)$.
- ii) ... the marginal revenue function $R'(x)$.
- iii) ... the marginal profit function $P'(x)$.
- a) $C(x) = (40x + 200)$ CHF $R(x) = 60x$ CHF
- b) $C(x) = (5x^2 + 20x + 100)$ CHF $R(x) = (-2x^2 + 100x)$ CHF
- c) $C(x) = (20x^2 + 50 + 3e^{4x})$ CHF $R(x) = (200x - e^{-4x^2})$ CHF

R2.4 Determine all positions where the given function has ...

- i) ... a local maximum.
- ... a local minimum.
- ii) ... a point of inflection.
- a) $f(x) = 2x^3 - 9x^2 + 12x - 1$
- b) $f(x)$ as in R2.2 a)

R2.5 The total revenue function for a commodity or a service is given by

$$R(x) = (-0.01x^2 + 36x) \text{ CHF}$$

Determine the maximum revenue if production is limited to at most 1500 units.

R2.6 If the total cost function for a commodity or a service is

$$C(x) = (x^2 + 100) \text{ CHF}$$

producing or rendering how many units x will result in a minimum average cost?
Determine that minimum average cost.

R2.7 A firm can produce 1000 units per month only. The monthly total cost is given by

$$C(x) = (200x + 300) \text{ CHF}$$

where x is the number produced. The total revenue is given by

$$R(x) = \left(-\frac{1}{100}x^2 + 250x\right) \text{ CHF}$$

How many items should the firm produce for a maximum profit?
Determine that maximum profit.

R2.8 Determine the indefinite integrals below:

a) $\int (x^4 - 3x^3 - 6) \, dx$

b) $\int \left(\frac{1}{2}x^6 - \frac{2}{3x^4}\right) \, dx$

R2.9 The equation of the third derivative f''' of a function f is given as follows:

$$f'''(x) = 3x + 1$$

Determine the equation of the function f such that $f''(0) = 0$, $f'(0) = 1$, $f(0) = 2$

R2.10 The marginal cost for producing a product or rendering a service is $C'(x) = (5x + 10)$ CHF, with a fixed cost of 800 CHF.

What will be the cost of producing or rendering 20 units?

R2.11 A certain firm's marginal cost $C'(x)$ and the derivative of the average revenue $\bar{R}'(x)$ are given as follows:

$$C'(x) = (6x + 60) \text{ CHF}$$

$$\bar{R}'(x) = -1 \text{ CHF}$$

If 10 items are produced or rendered, the total costs are 1000 CHF, and the revenue is 1700 CHF.

How many units will result in a maximum profit?
Determine that maximum profit.

R2.12 The supply function for a product or service is

$$p = f_s(x) = (4x + 4) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-x^2 + 49) \text{ CHF}$$

Determine the equilibrium point and both the consumer's and the producer's surplus there.

R2.13 (see next page)

R2.13 The supply function for a product or a service is

$$p = f_s(x) = \left(ax^2 - \frac{6}{5}x + 2 \right) \text{ CHF}$$

and the demand function is

$$p = f_d(x) = (-bx^2 + 110) \text{ CHF}$$

with unknown parameters a and b . The equilibrium price is 10 CHF, and the producer's surplus is 73.33 CHF (rounded).

Determine the two unknown parameters a and b .

Hint:

- Use the unrounded value $\left(73 + \frac{1}{3} \right) \text{ CHF} = \frac{220}{3} \text{ CHF}$ for the producer's surplus.