

Exercises 11

Derivative

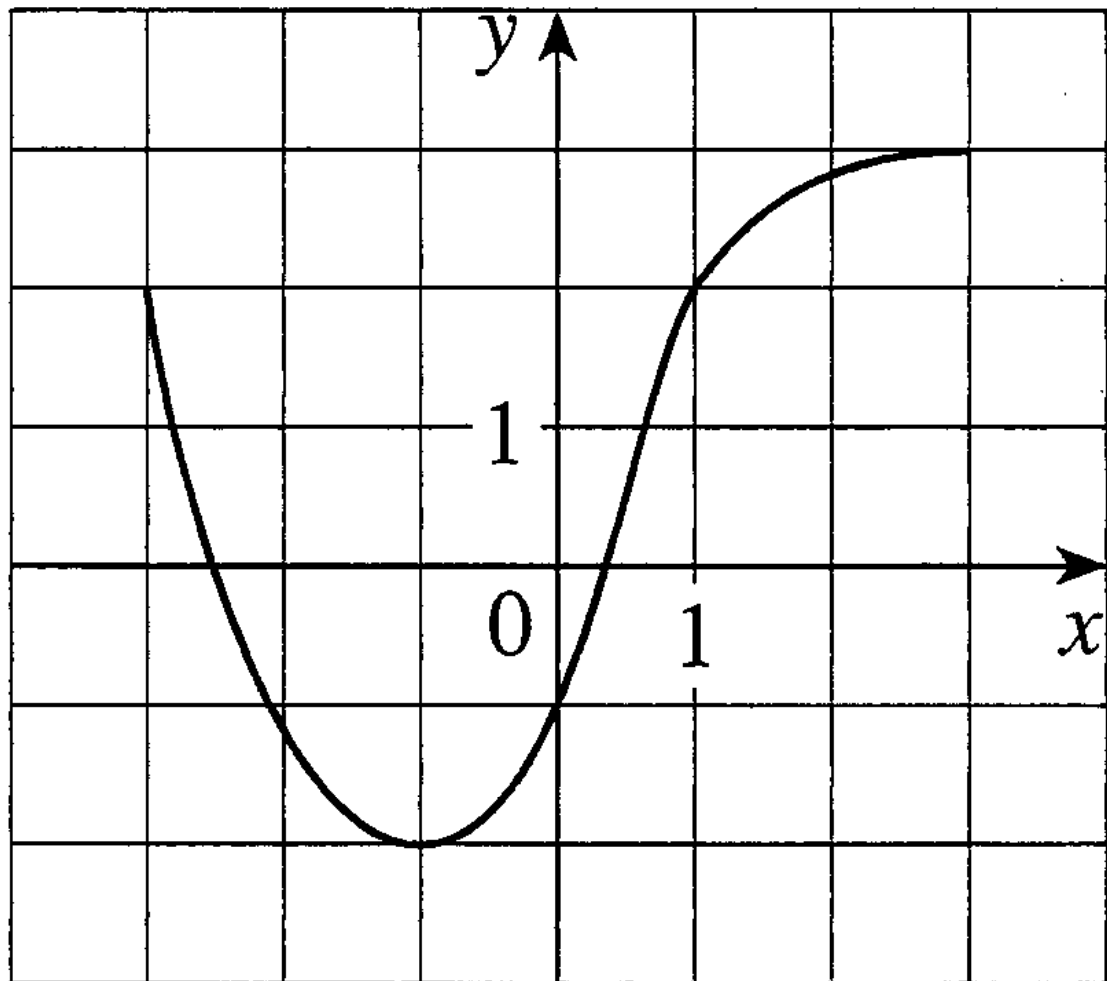
Derivative (rate of change), derivative (derived function) of constant/power/exponential functions

Objectives

- be able to estimate a derivative (rate of change) out of the graph of a function.
- be able to state the derivative (rate of change) of a constant and a linear function.
- be able to determine the derivative (derived function) of a constant and a linear function.
- be able to determine the derivative (derived function) of a basic power and a basic exponential function.
- be able to determine a derivative (rate of change) of a basic power and a basic exponential function.

Problems

11.1 The graph of a function f is given as follows:



Estimate the derivative (rate of change) $f'(x_0)$ at the given position x_0 :

- | | |
|---------------|---------------|
| a) $x_0 = -1$ | b) $x_0 = 0$ |
| c) $x_0 = 1$ | d) $x_0 = -2$ |

Hints:

- Draw the tangent to the graph of f at the given position x_0 .
- Choose any two points on the tangent, and estimate their coordinates.
- Determine the slope of the tangent out of the estimated coordinates of the two points.

11.2 For each of the following functions $f: \mathbb{R} \rightarrow \mathbb{R}$, $x \mapsto y = f(x) = \dots$

- i) ... draw the graph of f .
 - ii) ... state the derivative (rate of change) $f'(x_0)$ at the given position x_0 .
- a) $f(x) = 3$ $x_0 = 2$
 - b) $f(x) = c$ ($c \in \mathbb{R}$) any $x_0 \in \mathbb{R}$
 - c) $f(x) = 2x - 3$ $x_0 = 4$
 - d) $f(x) = mx + q$ ($m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R}$) any $x_0 \in \mathbb{R}$

Hint:

- If the graph of a function f is a straight line, the derivative (rate of change) $f'(x_0)$ is the slope of that straight line, i.e. $f'(x_0)$ has the same value at each position x_0 , and therefore does not depend on x_0 .

11.3 Determine $f'(x)$:

- | | | |
|-------------------------|---------------------------|--|
| a) $f(x) = 3$ | b) $f(x) = 0$ | c) $f(x) = -1$ |
| d) $f(x) = x^3$ | e) $f(x) = x^4$ | f) $f(x) = x^5$ |
| g) $f(x) = x^{17}$ | h) $f(x) = x^{200}$ | i) $f(x) = x^{100'001}$ |
| j) $f(x) = x^{-1}$ | k) $f(x) = x^{-2}$ | l) $f(x) = x^{-17}$ |
| m) $f(x) = \frac{1}{x}$ | n) $f(x) = \frac{1}{x^3}$ | o) $f(x) = \frac{1}{x^{99}}$ |
| p) $f(x) = 3^x$ | q) $f(x) = 5^x$ | r) $f(x) = \left(\frac{2}{3}\right)^x$ |

11.4 Determine the derivative (rate of change) $f'(x_0)$ of the function f at the indicated position x_0 :

- | | | | |
|--|---------------|--------------------------|---------------------------|
| a) $f(x) = x$ | i) $x_0 = 0$ | ii) $x_0 = 1$ | iii) $x_0 = -2$ |
| b) $f(x) = x^5$ | i) $x_0 = 0$ | ii) $x_0 = 2$ | iii) $x_0 = -\frac{2}{3}$ |
| c) $f(x) = x^{-4}$ | i) $x_0 = -1$ | ii) $x_0 = -\frac{4}{3}$ | iii) $x_0 = 0$ |
| d) $f(x) = \left(\frac{2}{3}\right)^x$ | i) $x_0 = 0$ | ii) $x_0 = 1$ | iii) $x_0 = -2$ |

11.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

- a) The derivative (rate of change) of a function f at the position x_0 is a ...
 - ☐ ... real number.
 - ☐ ... function.
 - ☐ ... tangent.
 - ☐ ... graph.
- b) (see next page)

b) The derivative (derived function) f' of a function f is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... tangent.
- ☐ ... graph.

c) $f'(x_0)$ is the slope of the ...

- ☐ ... secant through the points $(0|0)$ and $(x_0|f(x_0))$.
- ☐ ... secant through the points $(x_0+\Delta x|f(x_0+\Delta x))$ and $(x_0|f(x_0))$.
- ☐ ... tangent to the graph of f through $(x_0|f(x_0))$.
- ☐ ... tangent to the graph of f' through $(x_0|f(x_0))$.

Answers

- 11.1 a) $f'(-1) \approx 0$
b) $f'(0) \approx 2$
c) $f'(1) \approx \frac{3}{2}$
d) $f'(-2) \approx -\frac{5}{3}$
- 11.2 a) i) ...
ii) $f'(2) = 0$
b) i) ...
ii) $f'(x_0) = 0$ at any position x_0
c) i) ...
ii) $f'(4) = 2$
d) i) ...
ii) $f'(x_0) = m$ at any position x_0
- 11.3 a) $f'(x) = 0$ b) $f'(x) = 0$ c) $f'(x) = 0$
d) $f'(x) = 3x^2$ e) $f'(x) = 4x^3$ f) $f'(x) = 5x^4$
g) $f'(x) = 17x^{16}$ h) $f'(x) = 200x^{199}$ i) $f'(x) = 100'001x^{100'000}$
j) $f'(x) = -x^{-2}$ k) $f'(x) = -2x^{-3}$ l) $f'(x) = -17x^{-18}$
m) $f'(x) = -\frac{1}{x^2}$ n) $f'(x) = -\frac{3}{x^4}$ o) $f'(x) = -\frac{99}{x^{100}}$
p) $f'(x) = 3^x \ln(3)$ q) $f'(x) = 5^x \ln(5)$ r) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$
- 11.4 a) $f'(x) = 1$
i) $f'(0) = 1$ ii) $f'(1) = 1$ iii) $f'(-2) = 1$
b) $f'(x) = 5x^4$
i) $f'(0) = 0$ ii) $f'(2) = 80$ iii) $f'\left(-\frac{2}{3}\right) = \frac{80}{81}$
c) $f'(x) = -\frac{4}{x^5}$
i) $f'(-1) = 4$ ii) $f'\left(-\frac{4}{3}\right) = \frac{243}{256}$ iii) $f'(0)$ is not defined (division by zero)
d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$
i) $f'(0) = \ln\left(\frac{2}{3}\right)$ ii) $f'(1) = \frac{2}{3} \ln\left(\frac{2}{3}\right)$ iii) $f'(-2) = \frac{9}{4} \ln\left(\frac{2}{3}\right)$
- 11.5 a) 1st statement
b) 2nd statement
c) 3rd statement