

Exercises 7

Quadratic function and equations Quadratic function/equations, supply, demand, market equilibrium

Objectives

- know and understand the relation between a quadratic function and a quadratic equation.
- be able to solve a quadratic equation with the method of completing the square.
- be able to solve a quadratic equation by applying the quadratic formula.
- be able to solve special quadratic equations without applying the quadratic formula.
- be able to solve a quadratic equation containing a parameter.
- be able to determine the vertex form of the equation of a quadratic function out of the coordinates of the vertex and the coordinates of another point of the corresponding parabola.
- be able to determine the general form of the equation of a quadratic function out of the coordinates of three points of the corresponding parabola.
- be able to treat applied tasks in economics by means of quadratic equations or systems of quadratic equations.

Problems

7.1 Each quadratic equation can be converted into the following general form:

$$ax^2 + bx + c = 0 \quad (a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}, c \in \mathbb{R}) \quad (*)$$

Determine the number of solutions that a quadratic equation can have, i.e. try to find out the different possible cases of the number of solutions.

Hints:

- Remember our discussion about the possible number of solutions of a linear equation.
- Compare the left hand side of the quadratic equation (*) with the general form of the equation of a quadratic function.
- Think of the graph of a quadratic function.

7.2 Solve the quadratic equations below using ...

- i) ... the method of completing the square.
- ii) ... the quadratic formula.

State the solution set for each equation.

a) $x^2 + 10x + 24 = 0$

b) $2x^2 - 7x + 3 = 0$

c) $x^2 + 2x + 8 = 0$

d) $x^2 - 14x + 49 = 0$

7.3 Solve the quadratic equations below using the quadratic formula. State the solution set for each equation.

a) $x^2 + 22x + 121 = 0$

b) $5x^2 + 8x - 4 = 0$

c) $5x^2 - 8x + 4 = 0$

d) $24x^2 - 65x + 44 = 0$

e) $\frac{1}{6}x^2 - \frac{5}{4}x + \frac{3}{2} = 0$

f) $-9x^2 - 54x - 63 = 0$

7.4 Solve the equations below. State the solution set for each equation.

a) $9(x - 10) - x(x - 15) = x$

b) $3(x^2 + 2) - x(x + 9) = 11$

c) $y^3 + 19 = (y + 4)^3$

d) $\frac{9x - 8}{4x + 7} = \frac{3x}{2x + 5}$

e) $\frac{x^2}{x - 6} - \frac{6x}{6 - x} = 1$

f) $\frac{8}{x^2 - 4} + \frac{2}{2 - x} = 3x - 1$

7.5 Solve the quadratic equations below without using the quadratic formula.
State the solution set for each equation.

a) $(x + 2)(x + 5) = 0$

b) $(x - 8)(5x - 9) = 0$

c) $x^2 - 3x = 0$

d) $x^2 + 7x = 0$

e) $4x^2 - 9 = 0$

f) $100x^2 - 1 = 0$

g) $3x^2 = 27$

h) $x^2 = x$

7.6 Solve the equations below. State the solution set for each equation.

a) $(7 + x)(7 - x) = (3x + 2)^2 - (2x + 3)^2$

b) $(x - 3)(2x - 7) = 1$

c) $\frac{x-4}{x-5} = \frac{30-x^2}{x^2-5x}$

d) $\frac{x^2-x-2}{2-x} = 1$

e) $\frac{x^2-4}{x^2-4} = 0$

f) $\frac{x^2-4}{x^2-4} = 1$

7.7 The quadratic equations below contain a parameter p . Therefore, the solution set of the equations will depend on the value of this parameter.

Solve the equations for x .

a) $x^2 + x + p = 0$

b) $3x^2 + px - p = 0$

7.8 A parabola has the vertex V and contains the point P .

Determine the equation of the corresponding quadratic function both in the vertex and in the general form.

a) $V(2|4)$ $P(-1|7)$

b) $V(1|-8)$ $P(2|-7)$

7.9 A parabola contains the three points P , Q , and R .

Determine the equation of the corresponding quadratic function in the general form.

a) $P(-4|8)$ $Q(0|0)$ $R(10|15)$

b) $P(1|-1)$ $Q(2|4)$ $R(4|8)$

7.10 Find the equilibrium quantity and equilibrium price of a service for the given supply and demand functions f_s and f_d :

a) supply $p = f_s(q) = \left(\frac{1}{4}q^2 + 10\right)$ CHF
demand $p = f_d(q) = (86 - 6q - 3q^2)$ CHF

b) supply $p = f_s(q) = (q^2 + 8q + 16)$ CHF
demand $p = f_d(q) = (-3q^2 + 6q + 436)$ CHF

7.11 The total costs $C(x)$ for producing x items and the revenues $R(x)$ for selling x items are given by

$$C(x) = (2000 + 40x + x^2) \text{ CHF}$$

$$R(x) = 130x \text{ CHF}$$

Find the break-even values of x .

- 7.12 The total costs $C(x)$ for producing x items and the revenues $R(x)$ for selling x items are given by

$$C(x) = (x^2 + 100x + 80) \text{ CHF}$$

$$R(x) = (160x - 2x^2) \text{ CHF}$$

How many items are to be produced and sold in order to achieve a profit of 200 CHF?

- 7.13 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

- a) A quadratic equation ...

☐

... has no solution whenever the vertex of the graph of the corresponding quadratic function is below the x-axis.

☐

... always has one or two solutions.

☐

... has exactly one solution if the vertex of the graph of the corresponding quadratic function is on the x-axis.

☐

... can have infinitely many solutions.

- b) The graph of a quadratic function ...

☐

... is uniquely defined whenever the vertex and one further point of the graph are known.

☐

... is a straight line if the corresponding quadratic equation has exactly one solution.

☐

... is a quadratic equation.

☐

... can be determined by solving a quadratic equation.

- c) If the total cost function is quadratic and the revenue function is linear ...

☐

... the profit function is linear.

☐

... a break-even point corresponds to a solution of a quadratic equation.

☐

... there is always exactly one break-even point.

☐

... the profit can be positive at a break-even point.