

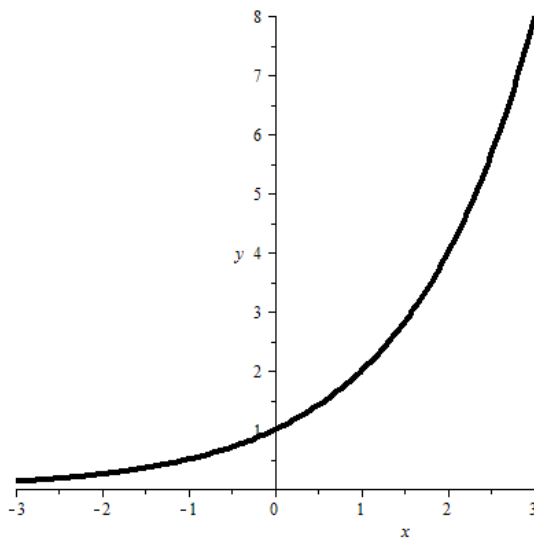
Exponential function

Definition

f: $D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \mapsto y = f(x) = c \cdot a^x$	$(a \in \mathbb{R}^+ \setminus \{1\}, c \in \mathbb{R} \setminus \{0\})$
$a > 1$: exponential growth	
$a < 1$: exponential decay	

Graph

1. $y = f(x) = 2^x$ ($c = 1, a = 2$)

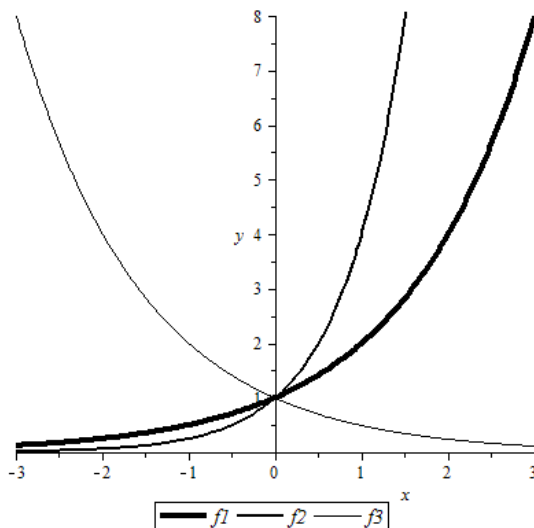


2. Parameter **a** (**varying a, keeping c constant**)

$y = f_1(x) = 2^x$ ($c = 1, a = 2$)

$y = f_2(x) = 4^x$ ($c = 1, a = 4$)

$y = f_3(x) = \left(\frac{1}{2}\right)^x$ ($c = 1, a = \frac{1}{2}$)

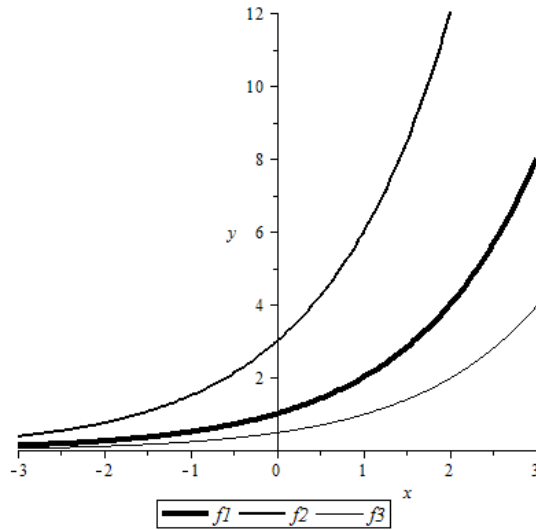


3. Parameter **c** (varying **c**, keeping **a** constant)

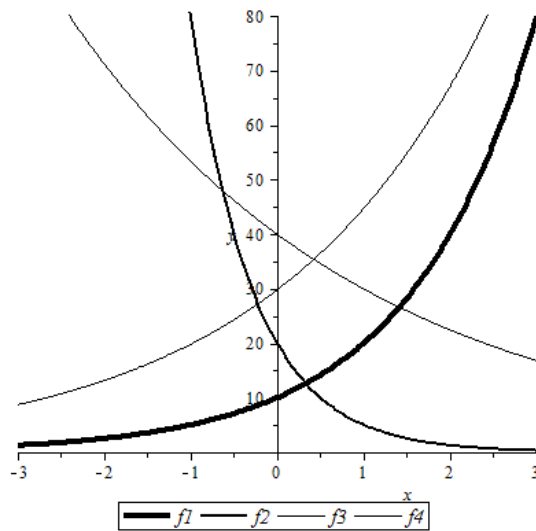
$y = f_1(x) = 2^x$ (**c = 1**, **a = 2**)

$y = f_2(x) = 3 \cdot 2^x$ (**c = 3**, **a = 2**)

$y = f_3(x) = \frac{1}{2} \cdot 2^x$ (**c = $\frac{1}{2}$** , **a = 2**)



4. $y = f_1(x) = 10 \cdot 2^x$ (**c = 10**, **a = 2**)
 $y = f_2(x) = 20 \cdot 0.25^x$ (**c = 20**, **a = 0.25**)
 $y = f_3(x) = 40 \cdot 0.75^x$ (**c = 40**, **a = 0.75**)
 $y = f_4(x) = 30 \cdot 1.5^x$ (**c = 30**, **a = 1.5**)



Examples

1. Compound interest (exponential **growth**)

$$C_n = C_0 \cdot q^n$$

C_0 = initial capital
 C_n = capital after n compounding periods
 n = number of compounding periods (often: 1 compounding period = 1 year)
 q = interest/growth factor = $1 + r$ ($r > 0, q > 1$)
 r = interest rate per compounding period

Ex.: $C_0 := 1000 \text{ CHF}, r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n \text{ CHF}$

2. Consumer price index (exponential **decay**)

$$P(t) = P_0 \cdot q^t$$

P_0 = initial price / initial purchasing power
 $P(t)$ = price / purchasing power at time t (often: t = number of years)
 q = decay factor = $1 + r$ ($r < 0, q < 1$)

Ex.: $P_0 := 100 \text{ CHF}, r := -3\% = -0.03 \Rightarrow q = 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t \text{ CHF}$