Review exercises 1 Functions and equations

Problems

R1.1 Which of the following relations are functions? Explain your answers.

a) $f_1 : \mathbb{R}_0^+ \to \mathbb{R}^+, x \mapsto y = f_1(x) = \sqrt{x}$

b) $f_2 : \{2, 3, 4, ...\} \to \mathbb{N}, x \mapsto y = f_2(x) = x - 1$

c) D = Set of all Swiss cantons
C = Set of all Swiss towns and cities
$f_3 : D \to C, x \mapsto y = f_3(x)$ = capital of x

d) $f_4 : \{x: x \in \mathbb{R} \text{ and } x \geq 3\} \to \mathbb{R}, x \mapsto y = f_4(x) = \frac{1}{x^2 - 9}$

e) $f_5 : \mathbb{R}_0^+ \to \mathbb{R}, x \mapsto y = f_5(x) = \log_a(x)$

R1.2 If $f(x) = 9x - x^2$, find ...

a) ... f(0).

b) ... f(-3).

c) ... $\frac{f(x + h) - f(x)}{h}$ and simplify the expression.

R1.3 Solve the equations below, and state the solution sets:

a) $3x - 8 = 23$

b) $\frac{6}{3x - 5} = \frac{6}{2x + 3}$

c) $\frac{2x + 5}{x + 7} = \frac{1}{3} + \frac{x - 11}{2x + 14}$

R1.4 Solve the following equations for x, and state the solution sets.
Take into account that the parameters a and p can be any real numbers.

a) $ax = 60$

b) $(p - 1)px = p^2 - 1$

R1.5 Solve each system of equations, and state the solution sets:

a) $2x + y = 19$
$x - 2y = 12$

b) $6x + 3y = 1$
$y = -2x + 1$

R1.6 Determine the equation of the linear function whose graph ...

a) ... has slope 4 and intercept 2.

b) ... passes through (-2|1) and has slope $\frac{2}{5}$.

c) ... (see next page)

c) ... passes through (-2|7) and (6|-4).

d) ... passes through (1|6) and is parallel to y = 4x - 6.

R1.7 A certain product has the following supply and demand functions:

$p = f_s(q) = (4q + 5)$ CHF
$p = f_d(q) = (- 2q + 81)$ CHF

a) If the price is 53 CHF, how many units are supplied and how many are demanded?

b) Determine both the equilibrium quantity and the equilibrium price.

R1.8 The total cost and total revenue for a certain product are given by the following:

$C(x) = (38.80x + 4500)$ CHF
$R(x) = 61.30x$ CHF

a) Determine the fixed costs.

b) Determine the variable costs for producing 10 units.

c) Determine the number of units required to break even.

R1.9 The supply function and the demand function for a product are linear and are determined by the tables that follow. Determine the quantity and price that will give market equilibrium.

Supply function		Demand function	
Price	Quantity	Price	Quantity
100 CHF	200	200 CHF	200
200 CHF	400	100 CHF	400
300 CHF	600	0 CHF	600

R1.10 Determine the solution sets of the equations below:

a) $4x - 3x^2 = 0$

b) $3x^2 - 6x = 9$

c) $4x^2 + 25 = 0$

d) $\frac{1}{x} + 2x = \frac{1}{3} + \frac{x + 1}{x}$

e) $\frac{x - 4}{x - 5} = \frac{30 - x^2}{x^2 - 5x}$

R1.11 Determine the equation of the quadratic function whose graph ...

a) ... has the vertex (2|4) and passes through (3|3).

b) ... passes through (-3|-3), (0|3), and (3|0).

R1.12 The supply function for a product is given by $p = (q^2 + 300)$ CHF, and the demand is given by $p/\text{CHF} + q = 410$. Determine the equilibrium quantity and price.

R1.13 If total costs for a service (x = service quantity) are given by $C(x) = (1760 + 8x + 0.6x^2)$ CHF and total revenues are given by $R(x) = (100x - 0.4x^2)$ CHF, determine the service quantity at the break-even points.

R1.14 Determine the equation of the exponential function whose graph passes through P and Q.

a) P(0|1) Q(2|9)

b) P(1|20) Q(2|100)

R1.15 Evaluate each logarithm without using a calculator:

a) $\log_5(1)$

b) $\log_2(8)$

c) $\log_3\left(\frac{1}{3}\right)$

d) $\log_3(3^8)$

e) $e^{\ln(5)}$

f) $10^{\lg(3.15)}$

R1.16 If 8000 CHF is borrowed for 3 years at an annual simple interest rate of 12%, what is the future value of the loan at the end of the 3 years?

R1.17 Mary borrowed 2000 CHF from her parents and repaid them 2100 CHF after 9 months. What annual simple interest rate did she pay?

R1.18 How much summer earnings must a college student deposit on August 31 in order to have 3000 CHF for tuition and fees on December 31 of the same year, if the investement pays simple interest at an annual interest rate of 6%?

R1.19 If 1000 CHF is invested for 4 years at a nominal annual interest rate of 8%, compounded quarterly, how much interest will be earned?

R1.20 How much must one invest now in order to have 18'000 CHF in 4 years if the investment pays interest at a nominal annual interest rate of 5.4%, compounded monthly?

R1.21 In 2010 an African country had a population of 4.5 million. The population has been increasing at 4% per year. What will the population be in 2030 if the growth factor does not change?

R1.22 A company wants to have 250'000 CHF available in 4 1/2 years for new construction. How much must be deposited at the beginning of each quarter to reach this goal if the investement pays interest at a nominal annual interest rate of 10.2%, compounded quarterly?

R1.23 A retirement account that pays interest at a nominal annual interest rate of 6.8%, compounded semiannually, contains 488'000 CHF. How long can 40'000 CHF be withdrawn at the end of each half-year until the account balance is 0 CHF?

R1.24 (see next page)

R1.24 Three years from now, a couple plan to spend 4 months travelling in China, Japan, and Southeast Asia. When they take their trip, they would like to withdraw 5000 CHF at the beginning of each month to cover their expenses for that month. Starting now, how much must they deposit at the beginning of each month for the next 3 years so that the account will provide the money they want while they are travelling? Assume that such an account pays interest at a nominal annual interest rate of 6.6%, compounded monthly.

R1.25 Mr S. is obligated to pay 25'000 CHF at the end of each of the following 8 years to his divorced wife. As a result of a personal profit in his company, he is able to pay the whole sum at the end of the first year (instead of making 8 payments at the end of each year). What amount of money does he have to pay at the end of the first year if interest, compounded annually, has been fixed at an annual interest rate of 4.5%?

R1.26 Mr P. is thinking about an investement for his retirement. He would like to withdraw 8000 CHF from an account at the end of each year for 15 years starting at the end of the year in which he turns 60. He assumes that money pays interest at an annual interest rate of 2.5% throughout the 15 years.

a) He wants to save the money by making 30 constant payments at the end of each year until turning 55. How much must he pay in each year, if his bank pays him interest at an annual interest rate of 3%, compounded annually? Assume that the interest rate of 3% is valid until Mr P. turns 60.

b) Mr P. has won 40'000 CHF in a lottery! Would this amount be sufficient for his retirement scheme if he pays the money in at the end of the year in which he turns 25? Assume the same interest conditions as in a).