Exercises 14 — Indefinite integral
Antiderivative, indefinite integral, coefficient/sum rule

Objectives

- be able to determine an antiderivative and the indefinite integral of a constant, basic power, and basic exponential function.
- be able to apply the coefficient and sum rules to determine the indefinite integral of a function.
- be able to determine the cost, revenue, and profit functions if the marginal cost, marginal revenue, and marginal profit functions are known.

Problems

14.1 Determine the indefinite integrals below:

a) $\int x^2\,dx$ b) $\int x^3\,dx$

c) $\int x^{-5}\,dx$ d) $\int \frac{1}{x^2}\,dx$

e) $\int \frac{1}{x^4}\,dx$ f) $\int 4\,dx$

g) $\int (-7)\,dx$ h) $\int e^x\,dx$

i) $\int e^{3x}\,dx$ j) $\int e^{-x}\,dx$

14.2 Determine the indefinite integral of the following functions f:

a) $f(x) = x^5$ b) $f(x) = 3x^2$

c) $f(x) = x^3 + 2x^2 - 5$ d) $f(x) = \frac{x^5}{2} - \frac{2}{3x^2}$

e) $f(x) = \frac{1}{2}x^3 - 2x^2 + 4x - 5$ f) $f(x) = x^{10} - \frac{1}{2}x^3 - x$

14.3 Determine the equations of those two antiderivatives F_1 and F_2 of f which fulfil the stated conditions.

a) $f(x) = 10x^2 + x$ $F_1(0) = 3$ $F_2(0) = -1$

b) $f(x) = x^3 + 3x + 1$ $F_1(2) = 5$ $F_2(4) = -8$

14.4 Suppose that we know the equation of the derivative f ' of a function f:

$$f'(x) = 3x^2 - 50x + 250$$

Determine the equation of the function f, if …

a) … $f(0) = 500$.

b) … $f(10) = 2500$.

14.5 Suppose that we know the equation of the second derivative f " of a function f:

$$f''(x) = 2x - 1$$

Determine the equation of the function f such that $f'(2) = 4$ and $f(1) = -1$.

14.6 If the monthly marginal cost for a product or a service is C'(x) = (2x + 100) CHF, with fixed costs amounting to 200 CHF, determine the total cost function for a month.

14.7 If the marginal cost for a product or a service is $C'(x) = (4x + 2)$ CHF, and the production or rendering of 10 units results in a total cost of 300 CHF, determine the total cost function.

14.8 If the marginal cost for a product or a service is $C'(x) = (4x + 40)$ CHF, and the total cost of producing or rendering 25 units is 3000 CHF, what will be the total cost for 30 units?

14.9 A firm knows that its marginal cost for a service is $C'(x) = (3x + 20)$ CHF, that its marginal revenue is $R'(x) = (-5x + 44)$ CHF, and that the cost of rendering of 10 units is 370 CHF.

Determine the …

a) … profit function $P(x)$.

b) … number of units that results in a maximum profit.

Hint:
- The revenue R is zero if no unit is sold. Thus, $R(0) = 0$ CHF.

14.10 Suppose that the marginal revenue $R'(x)$ and the derivative of the average cost $\overline{C}'(x)$ of a company are given as follows:

$$R'(x) = 400 \text{ CHF}$$

$$\overline{C}'(x) = \left(\frac{2}{15}x - 11 - \frac{10'000}{x^2}\right) \text{ CHF}$$

Producing or rendering 15 units results in a total cost of 16'750 CHF.

Determine the …

a) … profit function $P(x)$.

b) … number of units that results in a maximum profit.

c) … maximum profit.

14.11 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) An antiderivative of a function is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... set of functions.
- ☐ ... graph.

b) The indefinite integral of a function is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... set of functions.
- ☐ ... graph.

c) If $f = g'$ then ...

- ☐ ... f is an antiderivative of g.
- ☐ ... g is an antiderivative of f.
- ☐ ... f is the indefinite integral of g.
- ☐ ... g is the indefinite integral of f.