Exercises 12 — Differentiation rules
Coefficient, sum, product, exponential function, higher-order derivatives

Objectives

- be able to apply the coefficient, sum, and product rules to determine the derivative of a function.
- be able to determine a higher-order derivative of a function.

Problems

12.1 Determine the derivative by applying the **coefficient rule**:

a)	$f(x) = 3x^5$	b)	$f(x) = -4x^3$	c)	$f(x) = -x^{10}$
d)	$f(x) = a \cdot x^3$	e)	$f(x) = n \cdot x^{n-1}$	f)	$f(x) = 9 \cdot 3^x$
g)	$s(t) = \frac{1}{2} g \cdot t^2$	h)	$S(T) = \alpha \cdot T^4$	i)	$C(x) = (-3x)^3$

12.2 Determine the derivative by applying the **sum rule**:

a)	$f(x) = x^5 + x^6$	b)	$f(x) = x^{10} - x^9$	c)	$f(x) = 1 + x + 3x^3$
d)	$f(x) = \frac{1}{4} x^4 + 3x^2 - 2$	e)	$f(x) = 3x^2(x - 2)$	f)	$f(x) = - 3x^8 + x^5 - 3x + 99$
g)	$f(x) = ax^2 + bx + c$	h)	$f(x) = 3(a^2 - 2ax + x^2)$	i)	$f(x) = \frac{x^3}{3} - \frac{3}{x^3}$
j)	$s(t) = s_0 + v_0 t + \frac{1}{2} g \cdot t^2$	k)	$V(r) = - \frac{a}{r} + \frac{b}{r^2}$	l)	$C(n) = C_0(1 + nr)$

Hint:
- In some problems, the coefficient rule is needed, too.

12.3 Determine the derivative by applying the **product rule**:

a)	$f(x) = x \cdot e^x$	b)	$f(x) = x^3 \cdot 3^x$
c)	$f(x) = - 2x^5(x - 1)$	d)	$f(x) = (2x - 1) \cdot e^x$
e)	$f(x) = (2x - 1)(- 3x^2 - x + 1)$	f)	$V(r) = e^r \left(a \cdot r^2 - \frac{b}{r^3}\right)$

Hint:
- In some problems, the coefficient and/or the sum rule(s) is/are needed, too.

12.4 Determine the derivative of the exponential functions below:

a)	$f(x) = e^{4x}$	b)	$f(x) = e^{-x}$
c)	$f(x) = e^{-x^2}$	d)	$f(x) = e^{x^2-2x+5}$

12.5 Determine the derivative of the functions below. Apply the appropriate differentiation rule(s). Simplify and factorise the derivative as far as possible:

a)	$f(x) = (x - 2)\, e^{2x}$	b)	$f(x) = (2 - x^2)\, e^{-x}$
c)	$f(x) = (3x^3 - 2x^2 + x - 1)\, e^{-2x}$	d)	$P(v) = av^2\, e^{-bv^2}$

12.6 (see next page)

12.6 Determine the derivatives (rates of change) below:

a) $f'(2)$ with function f in 12.1 b)

b) $s'(4)$ with function s in 12.1 g)

c) $f'(-1)$ with function f in 12.2 g)

d) $P'(1)$ with function P in 12.5 d)

12.7 Determine the second and third derivatives of the functions below. Simplify and factorise the higher-order derivatives as far as possible:

a) Function f in 12.1 a)

b) Function f in 12.2 g)

c) Function f in 12.3 a)

d) Function f in 12.4 c)

Hint:
- You have already determinded the first derivatives of the corresponding functions.

12.8 Determine the indicated higher-order derivatives:

a) $f''(-1)$ with function f in 12.1 a)

Hint:
- You have already determined $f''(x)$ in 12.7 a).

b) $f'''(2)$ with function f in 12.4 c)

Hint:
- You have already determined $f'''(x)$ in 12.7 d).

12.9 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The third derivative of a function is a ...

- ☐ ... constant function if the second derivative is a quadratic function.
- ☐ ... quadratic function if the second derivative is a linear function.
- ☐ ... linear function if the first derivative is a quadratic function.
- ☐ ... constant function if the first derivative is a quadratic function.

b) The derivative of a ...

- ☐ ... product is the product of the derivatives of the single factors.
- ☐ ... product is the sum of the derivatives of the single factors.
- ☐ ... sum is the sum of the derivatives of the single addends.
- ☐ ... constant is the constant itself.

c) If $f(x) = c \cdot g(x) \cdot h(x)$ then $f'(x) = ...$

- ☐ ... 0
- ☐ ... $c \cdot g'(x) \cdot h'(x)$
- ☐ ... $c \cdot g(x) \cdot h'(x) + c \cdot g'(x) \cdot h(x)$
- ☐ ... $c \cdot g'(x) \cdot h'(x) + c \cdot g(x) \cdot h(x)$