Exercises 4 — Linear function and equations
Linear equations

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to perform a case differentiation.
- be able to treat applied tasks in economics by means of linear equations.

Problems

4.1 Determine the solution sets of the following equations:

a) $19x - 32 + 17x = 18x - 30 + 16x - 4$

b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$

c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$

d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

4.2 Determine the solution sets of the following equations:

a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$

b) $184 - 6(x - 24) = 214 - 3(2x - 38)$

c) $(x - 5)(x - 2) = (x - 4)(x - 3)$

d) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

4.3 Determine the solution sets of the following equations:

a) $\dfrac{x + 3}{5} = \dfrac{2x - 8}{3}$

b) $\dfrac{x + 3}{4} + \dfrac{1 - 3x}{7} = 0$

c) $\dfrac{2}{x - 1} = \dfrac{1}{x - 2}$

d) $\dfrac{x}{x - 1} = \dfrac{x - 1}{x - 2}$

4.4 The equations below are equations in the variable x. Furthermore, the equations contain parameters a and b. Therefore, the solution sets of the equations depend on the values of those parameters.

Solve the equations for x, and determine the solution sets.
Take into account that the parameters a and b can be any real numbers.

a) $x(a - 3) = a$

   Hints:
   - You may want to divide both sides of the equation by a - 3. However, this is not allowed if a - 3 = 0, i.e. if a = 3, as dividing by 0 is not defined.
   - When solving the equation, consider the two cases a ≠ 3 and a = 3.

b) $(x + 1)(b - 2) = 2bx$

c) $(a - b)x = a$

4.5   The graph of a linear function f with slope a contains the point P. Find the equation of the linear function.

   a)   a = -5   P(5|-3)

   b)   a = 2   P(3|0)

   c)   a = 0   P(2|3)

4.6   Alps Bikes uses the formula B(t) = (- 400 t + 5000) CHF to find the book value B(t) of a mountain bike after the time t after its purchase (t = number of years after the purchase).

   a)   What do the numbers - 400 and 5000 signify?

   b)   How long will it take the mountain bike to depreciate completely?

4.7   Two items A and B depreciate linearly:

           Item A   original value = 200 CHF
                           depreciation = 16 CHF/year

           Item B   original value = 240 CHF
                           depreciation = 32 CHF/year

   a)   How long will it take the two items to depreciate completely?

   b)   After how much time will the two items have the same value?

   Hint:
   - Think of the temporal development of the values as linear functions.

4.8   Decide which statements are true or false. Put a mark into the corresponding box.
        In each problem a) to c), exactly one statement is true.

   a)   The solution set of a linear equation ...

           ☐   ... always contains at least one element.
           ☐   ... never contains two elements.
           ☐   ... only contains elements if the linear equation corresponds to a constant function.
           ☐   ... cannot be the empty set.

   b)   If a linear equation has exactly one solution ...

           ☐   ... the graph of the corresponding linear function intersects the x-axis.
           ☐   ... the equation does not contain any parameters.
           ☐   ... the solution must be an integer.
           ☐   ... no other linear equation can have the same solution.

   c)   If a linear equation has the solution x = 2, it can be concluded that ...

           ☐   ... x = 3 is not a solution.
           ☐   ... the graph of the corresponding linear function intersects the x-axis at x = 2.
           ☐   ... P(2|0) is a point of the graph of the corresponding linear function.
           ☐   ... P(0|2) is a point of the graph of the corresponding linear function.