Exercises 1 Number sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, set operations

Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what a set, an element of a set, an empty set, a subset, an intersection, a union, and a set difference are.
- be able to perform basic set operations.

Problems

1.1 Decide whether each statement is true or false:

a)	$4 \in \mathbb{N}$	b)	$-\frac{14}{7} \in \mathbb{Z}$	c)	$\sqrt{2} \in \mathbb{Q}$
d)	$\sqrt{9} \in \mathbb{N}$	e)	$\sqrt{9} \in \mathbb{Q}$	f)	$\sqrt{9} \in \mathbb{R}$
g)	$1.67854 \in \mathbb{Q}$	h)	$1.67\overline{854} \in \mathbb{Q}$	i)	$\mathbb{N} \subset \mathbb{Z}$
j)	$\mathbb{Z} \subset \mathbb{Q}$	k)	$\mathbb{Q} \subset \mathbb{R}$	l)	$\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$

1.2 Determine the following sets:

a)	$\mathbb{Z} \setminus \mathbb{N}$	b)	$\mathbb{Z} \cup \mathbb{N}$	c)	$\mathbb{Z} \cap \mathbb{N}$
d)	$\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$	e)	$\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$	f)	$(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$

1.3 Look at the sets A, B, and C:

A = Set of all cities of the world
B = Set of all European cities
C = Set of all coastal cities of the world

Find at least five elements of the following sets:

a)	$B \cap C$	b)	$B \setminus C$
c)	$C \setminus B$	d)	$A \setminus (B \cup C)$

1.4 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a)
- ☐ $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$
- ☐ $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$
- ☐ $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$
- ☐ $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, ...\}$

b) A = Set of all cities of the world
B = Set of all European cities

- ☐ $A \cap B = A$
- ☐ $A \cup B = B$
- ☐ $B \in A$
- ☐ $B \subset A$

c) (see next page)

c) Assume that x is a rational number. Therefore, it can be concluded that x is ...

- ☐ ... a real number.
- ☐ ... an integer.
- ☐ ... a fraction where both numerator and denominator are natural numbers.
- ☐ ... a natural number.

Answers

1.1	a)	true	b)	true	c)	false
	d)	true	e)	true	f)	true
	g)	true	h)	true	i)	true
	j)	true	k)	true	l)	false

1.2 a) $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, ...\}$

b) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$

c) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$

d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$

e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$

f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$

1.3 a) $B \cap C$ = {Lisbon, Copenhagen, Barcelona, Naples, Stockholm, ...}

b) $B \setminus C$ = {London, Paris, Madrid, Berlin, Rome, ...}

c) $C \setminus B$ = {Tokyo, San Francisco, Sydney, Rio de Janeiro, Cape Town, ...}

d) $A \setminus (B \cup C)$ = {Chicago, Mexico City, Nairobi, Beijing, Bogotá, ...}

1.4 a) 3rd statement

b) 4th statement

c) 1st statement