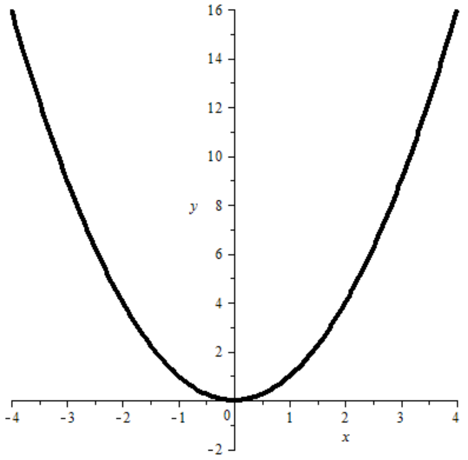
Quadratic function

Definition

$f\colon D \to \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \mapsto y = f(x) = ax^2 + bx + c$ general form	$(a \in \mathbb{R}\backslash\{0\}, b \in \mathbb{R}, c \in \mathbb{R})$
$y = f(x) = a(x - u)^2 + v$ vertex form	$(a \in \mathbb{R}\backslash\{0\}, u \in \mathbb{R}, v \in \mathbb{R})$

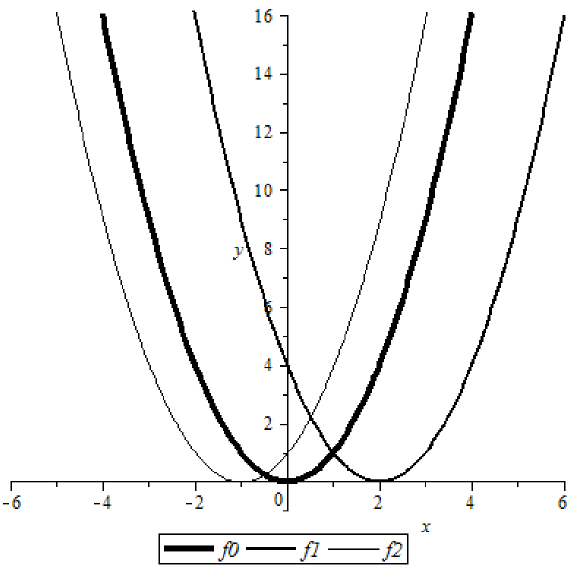
Graph

1. $y = f(x) = x^2$ $(a = 1, u = 0, v = 0)$



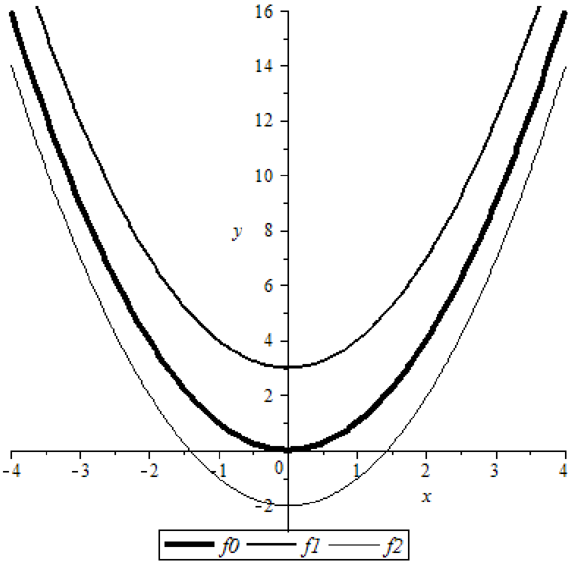
2. Parameter **u** (**varying u**, keeping a and v constant)

$y = f_0(x) = x^2$ $(a = 1, \mathbf{u = 0}, v = 0)$
$y = f_1(x) = (x - 2)^2$ $(a = 1, \mathbf{u = 2}, v = 0)$
$y = f_2(x) = (x + 1)^2$ $(a = 1, \mathbf{u = -1}, v = 0)$



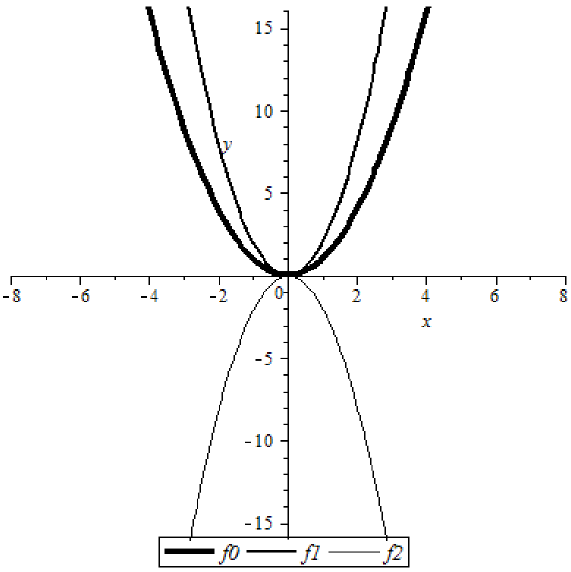
3. Parameter **v** (**varying v**, keeping a and u constant)

$y = f_0(x) = x^2$ (a = 1, u = 0, **v = 0**)
$y = f_1(x) = x^2 + 3$ (a = 1, u = 0, **v = 3**)
$y = f_2(x) = x^2 – 2$ (a = 1, u = 0, **v = -2**)



4. Parameter **a** (**varying a**, keeping u and v constant)

$y = f_0(x) = x^2$ (**a = 1**, u = 0, v = 0)
$y = f_1(x) = 2x^2$ (**a = 2**, u = 0, v = 0)
$y = f_2(x) = -2x^2$ (**a = -2**, u = 0, v = 0)

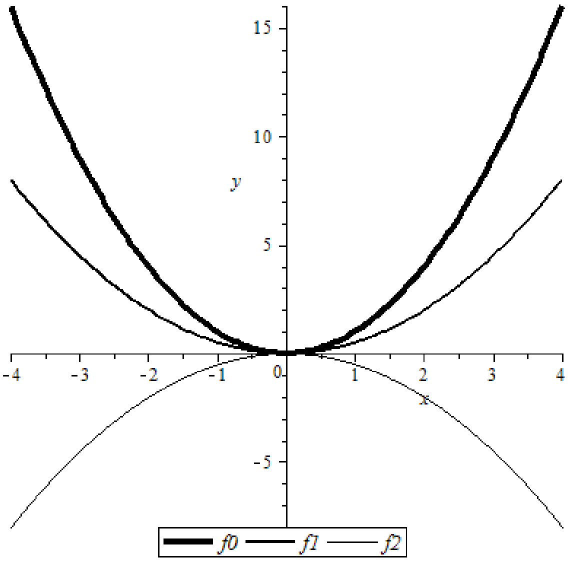


5. Parameter **a** (**varying a**, keeping u and v constant)

$y = f_0(x) = x^2$ $(\mathbf{a = 1}, u = 0, v = 0)$

$y = f_1(x) = \frac{1}{2}x^2$ $\left(\mathbf{a = \frac{1}{2}}, u = 0, v = 0\right)$

$y = f_2(x) = -\frac{1}{2}x^2$ $\left(\mathbf{a = -\frac{1}{2}}, u = 0, v = 0\right)$



6. The **graph** of a quadratic function is a **parabola**.

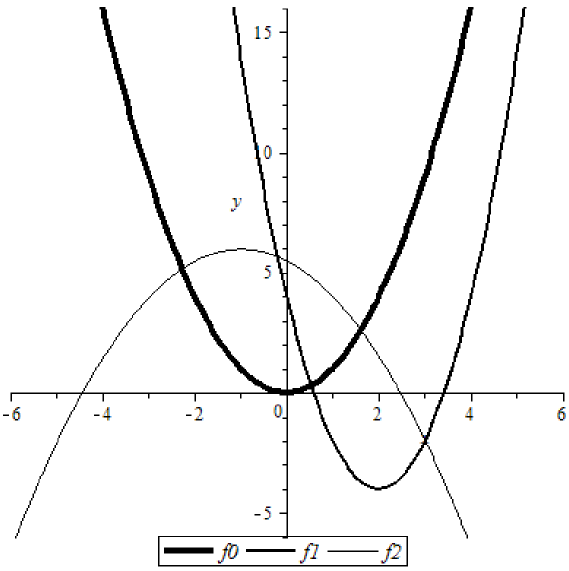
The parameter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** V of the parabola: V(u|v)

$y = f_0(x) = x^2$ $(a = 1, u = 0, v = 0)$ V(0|0)

$y = f_1(x) = 2(x-2)^2 - 4$ $(a = 2, u = 2, v = -4)$ V(2|-4)

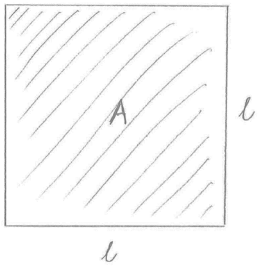
$y = f_2(x) = -\frac{1}{2}(x+1)^2 + 6$ $\left(a = -\frac{1}{2}, u = -1, v = 6\right)$ V(-1|6)



Examples

1. Nature/Physics: Trajectories of water in a fountain

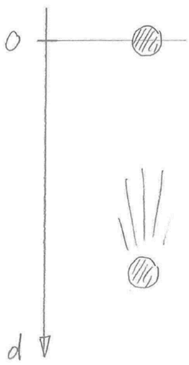
2. Geometry: Square



Area A for side length l: $A = l^2$

f: $\mathbb{R}^+ \to \mathbb{R}$
$l \mapsto A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t: $d = \frac{1}{2}gt^2$ (g = gravity field strength)

f: $\mathbb{R} \to \mathbb{R}$
$t \mapsto d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand