Exercises 15 Applications of differential calculus
Local/global maxima/minima, points of inflection

Objectives

- be able to determine the local maxima and minima of a function.
- be able to determine the points of inflection of a function.
- be able to determine the global maximum and the global minimum of a cost, revenue, and profit function.
- be able to determine the global minimum of an average cost, average revenue, and average profit function.

Problems

15.1 For each function, determine …

i) … all local maxima and minima.

ii) … all points of inflection.

a) $f(x) = x^2 - 4$

b) $f(x) = - 8x^3 + 12x^2 + 18x$

c) $s(t) = t^4 - 8t^2 + 16$

d) $f(x) = x\, e^{-x}$

e) * $f(x) = (1 - e^{-2x})^2$

f) * $V(r) = - D\left(\frac{2a}{r} - \frac{a^2}{r^2}\right)$ $(D > 0,\ a > 0)$

15.2 If the total profit for a commodity is

$$P(x) = (2000x + 20x^2 - x^3) \text{ CHF}$$

where x is the number of items sold, determine the level of sales, x, that maximises profit, and find the maximum profit.

Hints:
- First, find the local maxima.
- Then, check if one of the local maxima is the global maximum.

15.3 If the total cost for a service concerning a tourism event is given by

$$C(x) = \left(\frac{1}{4}x^2 + 4x + 100\right) \cdot 100 \text{ CHF}$$

where x represents the extent of the service, what value of x will result in a minimum average cost? Determine the minimum average cost.

15.4 Suppose that the production capacity for a certain commodity cannot exceed 30. If the total profit for this company is

$$P(x) = (4x^3 - 210x^2 + 3600x) \text{ CHF}$$

where x is the number of units sold, determine the number of items that will maximise profit.

15.5 (see next page)

15.5 Suppose the annual profit for a store is given by

$$P(x) = (-0.1x^3 + 3x^2) \cdot 1000 \text{ CHF}$$

where x is the number of years past 2010. If this model is accurate, determine the point of inflection for the profit.

15.6 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) If f has a local maximum at $x = x_0$ it can be concluded that ...

- ☐ ... $f(x_0) > f(x)$ for any $x \neq x_0$
- ☐ ... $f(x_0) > f(x)$ for any $x > x_0$
- ☐ ... $f(x_0) > f(x)$ for any $x < x_0$
- ☐ ... $f(x_0) > f(x)$ for all x which are in a certain neighbourhood of x_0

b) If $f(x_0) < 0$, $f'(x_0) = 0$, and $f''(x_0) \neq 0$, it can be concluded that f has ...

- ☐ ... no local minimum at $x = x_0$
- ☐ ... no local maximum at $x = x_0$
- ☐ ... no point of inflection at $x = x_0$
- ☐ ... a point of inflection at $x = x_0$

c) The global maximum of a function ...

- ☐ ... is always a local maximum.
- ☐ ... can be a local minimum.
- ☐ ... can be a local maximum.
- ☐ ... always exists.