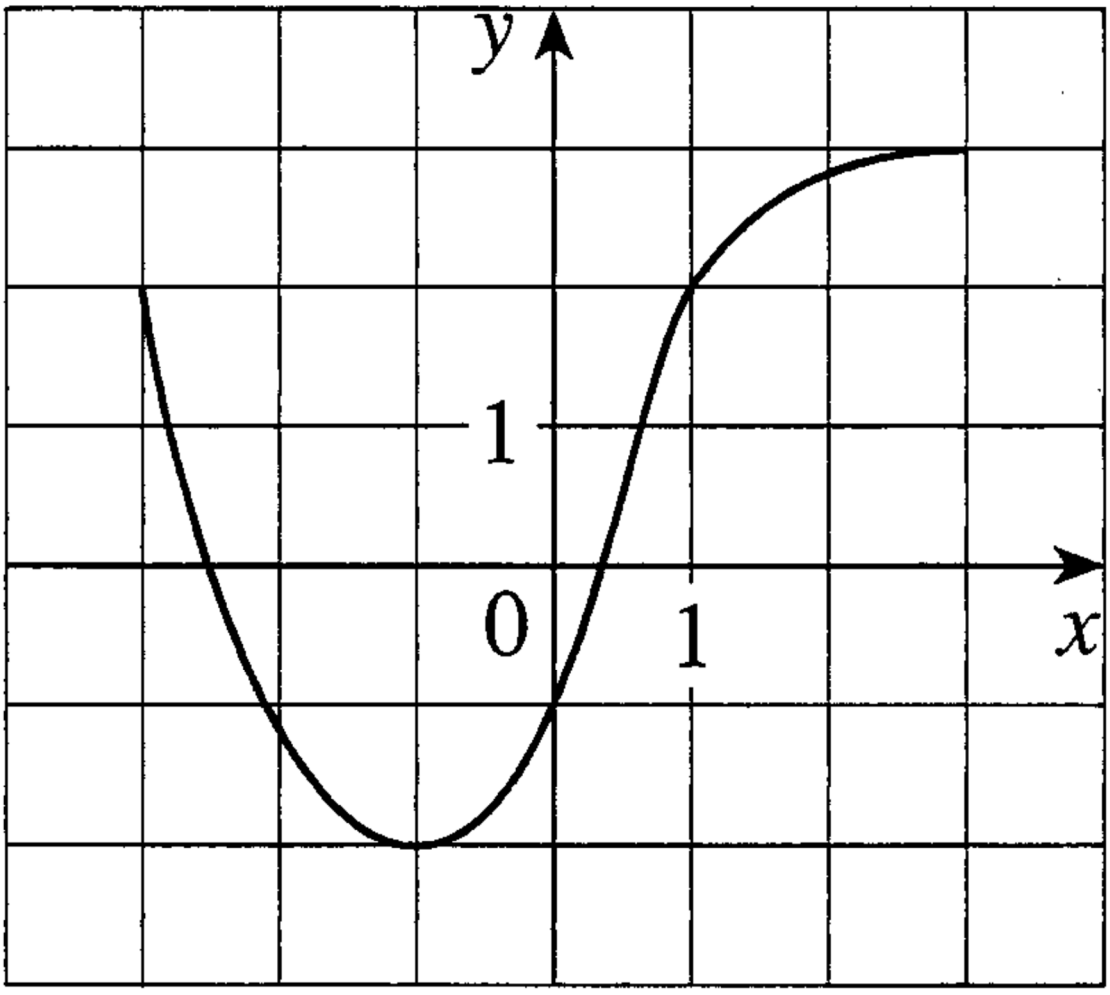
Exercises 13 Derivative
Derivative (rate of change), derivative (derived function) of constant/power/exponential functions

Objectives

- be able to estimate a derivative (rate of change) out of the graph of a function.
- be able to state the derivative (rate of change) of a constant and a linear function.
- be able to determine the derivative (derived function) of a constant and a linear function.
- be able to determine the derivative (derived function) of a basic power and a basic exponential function.
- be able to determine a derivative (rate of change) of a basic power and a basic exponential function.

Problems

13.1 The graph of a function f ist given as follows:



Estimate the derivative (rate of change) $f'(x_0)$ at the given position x_0:

a) $x_0 = -1$ b) $x_0 = 0$

c) $x_0 = 1$ d) $x_0 = -2$

Hints:
- Draw the tangent to the graph of f at the given position x_0.
- Choose any two points on the tangent, and estimate their coordinates.
- Determine the slope of the tangent out of the estimated coordinates of the two points.

13.2 For each of the following functions f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = ...$

i) ... draw the graph of f.

ii) ... state the derivative (rate of change) $f'(x_0)$ at the given position x_0.

a)	$f(x) = 3$	$x_0 = 2$
b)	$f(x) = c \quad (c \in \mathbb{R})$	any $x_0 \in \mathbb{R}$
c)	$f(x) = 2x - 3$	$x_0 = 4$
d)	$f(x) = mx + q \quad (m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R})$	any $x_0 \in \mathbb{R}$

Hint:
- If the graph of a function f is a straight line, the derivative (rate of change) $f'(x_0)$ is the slope of that straight line, i.e $f'(x_0)$ has the same value at each position x_0, and therefore does not depend on x_0.

13.3 Determine f '(x):

a)	$f(x) = 3$	b)	$f(x) = 0$	c)	$f(x) = -1$
d)	$f(x) = x^3$	e)	$f(x) = x^4$	f)	$f(x) = x^5$
g)	$f(x) = x^{17}$	h)	$f(x) = x^{200}$	i)	$f(x) = x^{100'001}$
j)	$f(x) = x^{-1}$	k)	$f(x) = x^{-2}$	l)	$f(x) = x^{-17}$
m)	$f(x) = \frac{1}{x}$	n)	$f(x) = \frac{1}{x^3}$	o)	$f(x) = \frac{1}{x^{99}}$
p)	$f(x) = 3^x$	q)	$f(x) = 5^x$	r)	$f(x) = \left(\frac{2}{3}\right)^x$

13.4 Determine the derivative (rate of change) $f'(x_0)$ of the function f at the indicated position x_0:

a) $f(x) = x$

i)	$x_0 = 0$	ii)	$x_0 = 1$	iii)	$x_0 = -2$

b) $f(x) = x^5$

i)	$x_0 = 0$	ii)	$x_0 = 2$	iii)	$x_0 = -\frac{2}{3}$

c) $f(x) = x^{-4}$

i)	$x_0 = -1$	ii)	$x_0 = -\frac{4}{3}$	iii)	$x_0 = 0$

d) $f(x) = \left(\frac{2}{3}\right)^x$

i)	$x_0 = 0$	ii)	$x_0 = 1$	iii)	$x_0 = -2$

13.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The derivative (rate of change) of a function f at the position x_0 is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... tangent.
- ☐ ... graph.

b) (see next page)

b) The derivative (derived function) f ' of a function f is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... tangent.
- ☐ ... graph.

c) $f'(x_0)$ is the slope of the ...

- ☐ ... secant through the points $(0|0)$ and $(x_0|f(x_0))$.
- ☐ ... secant through the points $(x_0+\Delta x|f(x_0+\Delta x))$ and $(x_0|f(x_0))$.
- ☐ ... tangent to the graph of f through $(x_0|f(x_0))$.
- ☐ ... tangent to the graph of f ' through $(x_0|f(x_0))$.