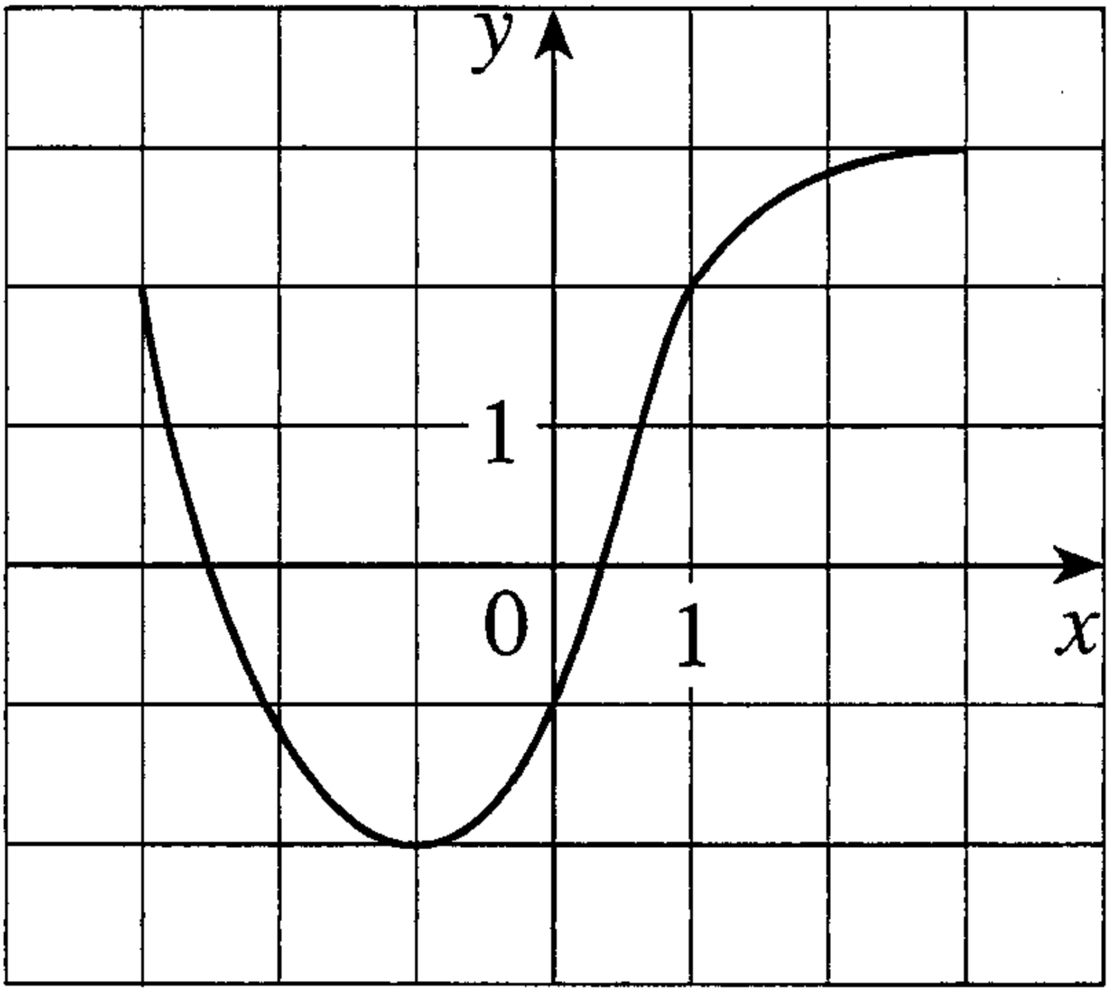
## Exercises 13 Derivative
## Derivative (rate of change), derivative (derived function) of constant/power/exponential functions

### Objectives

- be able to estimate a derivative (rate of change) out of the graph of a function.
- be able to state the derivative (rate of change) of a constant and a linear function.
- be able to determine the derivative (derived function) of a constant and a linear function.
- be able to determine the derivative (derived function) of a basic power and a basic exponential function.
- be able to determine a derivative (rate of change) of a basic power and a basic exponential function.

### Problems

13.1 The graph of a function f ist given as follows:




Estimate the derivative (rate of change) $f'(x_0)$ at the given position $x_0$:

a) $x_0 = -1$ b) $x_0 = 0$

c) $x_0 = 1$ d) $x_0 = -2$

Hints:
- Draw the tangent to the graph of f at the given position $x_0$.
- Choose any two points on the tangent, and estimate their coordinates.
- Determine the slope of the tangent out of the estimated coordinates of the two points.

13.2 For each of the following functions $f: \mathbb{R} \to \mathbb{R}, x \mapsto y = f(x) = ...$

i) ... draw the graph of f.

ii) ... state the derivative (rate of change) $f'(x_0)$ at the given position $x_0$.

a) $f(x) = 3$ $x_0 = 2$

b) $f(x) = c$ $(c \in \mathbb{R})$ any $x_0 \in \mathbb{R}$

c) $f(x) = 2x - 3$ $x_0 = 4$

d) $f(x) = mx + q$ $(m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R})$ any $x_0 \in \mathbb{R}$

Hint:
- If the graph of a function f is a straight line, the derivative (rate of change) $f'(x_0)$ is the slope of that straight line, i.e $f'(x_0)$ has the same value at each position $x_0$, and therefore does not depend on $x_0$.

13.3 Determine $f'(x)$:

a) $f(x) = 3$ b) $f(x) = 0$ c) $f(x) = -1$

d) $f(x) = x^3$ e) $f(x) = x^4$ f) $f(x) = x^5$

g) $f(x) = x^{17}$ h) $f(x) = x^{200}$ i) $f(x) = x^{100'001}$

j) $f(x) = x^{-1}$ k) $f(x) = x^{-2}$ l) $f(x) = x^{-17}$

m) $f(x) = \frac{1}{x}$ n) $f(x) = \frac{1}{x^3}$ o) $f(x) = \frac{1}{x^{99}}$

p) $f(x) = 3^x$ q) $f(x) = 5^x$ r) $f(x) = \left(\frac{2}{3}\right)^x$

13.4 Determine the derivative (rate of change) $f'(x_0)$ of the function f at the indicated position $x_0$:

a) $f(x) = x$

i) $x_0 = 0$ ii) $x_0 = 1$ iii) $x_0 = -2$

b) $f(x) = x^5$

i) $x_0 = 0$ ii) $x_0 = 2$ iii) $x_0 = -\frac{2}{3}$

c) $f(x) = x^{-4}$

i) $x_0 = -1$ ii) $x_0 = -\frac{4}{3}$ iii) $x_0 = 0$

d) $f(x) = \left(\frac{2}{3}\right)^x$

i) $x_0 = 0$ ii) $x_0 = 1$ iii) $x_0 = -2$

13.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The derivative (rate of change) of a function f at the position $x_0$ is a ...

- ☐ ... real number.
- ☐ ... function.
- ☐ ... tangent.
- ☐ ... graph.

b) (see next page)

b) The derivative (derived function) f ' of a function f is a ...

- [ ] ... real number.
- [ ] ... function.
- [ ] ... tangent.
- [ ] ... graph.

c) $f'(x_0)$ is the slope of the ...

- [ ] ... secant through the points $(0|0)$ and $(x_0|f(x_0))$.
- [ ] ... secant through the points $(x_0+\Delta x|f(x_0+\Delta x))$ and $(x_0|f(x_0))$.
- [ ] ... tangent to the graph of f through $(x_0|f(x_0))$.
- [ ] ... tangent to the graph of f ' through $(x_0|f(x_0))$.

**Answers**

13.1
a) $f'(-1) \approx 0$
b) $f'(0) \approx 2$
c) $f'(1) \approx \frac{3}{2}$
d) $f'(-2) \approx -\frac{5}{3}$

13.2
a) i) ...
ii) $f'(2) = 0$
b) i) ...
ii) $f'(x_0) = 0$ at any position $x_0$
c) i) ...
ii) $f'(4) = 2$
d) i) ...
ii) $f'(x_0) = m$ at any postion $x_0$

13.3

| | | | | | |
|---|---|---|---|---|---|
| a) | $f'(x) = 0$ | b) | $f'(x) = 0$ | c) | $f'(x) = 0$ |
| d) | $f'(x) = 3x^2$ | e) | $f'(x) = 4x^3$ | f) | $f'(x) = 5x^4$ |
| g) | $f'(x) = 17x^{16}$ | h) | $f'(x) = 200x^{199}$ | i) | $f'(x) = 100'001x^{100'000}$ |
| j) | $f'(x) = -x^{-2}$ | k) | $f'(x) = -2x^{-3}$ | l) | $f'(x) = -17x^{-18}$ |
| m) | $f'(x) = -\frac{1}{x^2}$ | n) | $f'(x) = -\frac{3}{x^4}$ | o) | $f'(x) = -\frac{99}{x^{100}}$ |
| p) | $f'(x) = 3^x \ln(3)$ | q) | $f'(x) = 5^x \ln(5)$ | r) | $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$ |

13.4
a) $f'(x) = 1$
i) $f'(0) = 1$ ii) $f'(1) = 1$ iii) $f'(-2) = 1$
b) $f'(x) = 5x^4$
i) $f'(0) = 0$ ii) $f'(2) = 80$ iii) $f'\left(-\frac{2}{3}\right) = \frac{80}{81}$
c) $f'(x) = -\frac{4}{x^5}$
i) $f'(-1) = 4$ ii) $f'\left(-\frac{4}{3}\right) = \frac{243}{256}$ iii) $f'(0)$ is not defined (division by zero)
d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$
i) $f'(0) = \ln\left(\frac{2}{3}\right)$ ii) $f'(1) = \frac{2}{3} \ln\left(\frac{2}{3}\right)$ iii) $f'(-2) = \frac{9}{4} \ln\left(\frac{2}{3}\right)$

13.5
a) 1st statement
b) 2nd statement
c) 3rd statement