

## Exercises 5                      Linear function and equations Linear equations

### Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to treat applied tasks in economics by means of linear equations.

### Problems

5.1 Determine the solution sets of the following equations:

- a)  $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b)  $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c)  $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d)  $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

5.2 Determine the solution sets of the following equations:

- a)  $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b)  $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c)  $(x - 5)(x - 2) = (x - 4)(x - 3)$
- d)  $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

5.3 Determine the solution sets of the following equations:

- a)  $\frac{x+3}{5} = \frac{2x-8}{3}$
- b)  $\frac{x+3}{4} + \frac{1-3x}{7} = 0$
- c)  $\frac{2}{x-1} = \frac{1}{x-2}$
- d)  $\frac{x}{x-1} = \frac{x-1}{x-2}$

5.4 The equations below are equations in the variable  $x$ . Furthermore, the equations contain parameters  $a$  and  $b$ . Therefore, the solution sets of the equations depend on the values of those parameters.

Solve the equations for  $x$ , and determine the solution sets.

Take into account that the parameters  $a$  and  $b$  can be any real numbers.

a)  $x(a - 3) = a$

Hints:

- You may want to divide both sides of the equation by  $a - 3$ . However, this is not allowed if  $a - 3 = 0$ , i.e. if  $a = 3$ , as dividing by 0 is not defined.
- Solve the equation for the two cases  $a \neq 3$  and  $a = 3$ .

b)  $(x + 1)(b - 2) = 2bx$

c)  $a(1 + x) - 1 = x(2a - 1)$

d)  $(a - b)x = a$

5.5 The graph of a linear function  $f$  with slope  $a$  contains the point  $P$ . Find the equation of the linear function.

- a)  $a = -5$                        $P(5|-3)$
- b)  $a = 2$                           $P(3|0)$
- c)  $a = 0$                           $P(2|3)$

5.6 Alps Bikes uses the formula  $B(t) = (-400t + 5000)$  CHF to find the book value  $B(t)$  of a mountain bike after the time  $t$  after its purchase ( $t$  = number of years after the purchase).

- a) What do the numbers  $-400$  and  $5000$  signify?
- b) How long will it take the mountain bike to depreciate completely?

5.7 Two items A and B depreciate linearly:

- Item A                      original value = 200 CHF  
                                    depreciation = 16 CHF/year
- Item B                      original value = 240 CHF  
                                    depreciation = 32 CHF/year

- a) How long will it take the two items to depreciate completely?
- b) After how much time will the two items have the same value?

Hint:

- Think of the temporal development of the values as linear functions.

5.8 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

- a) The solution set of a linear equation ...
  - ... always contains at least one element.
  - ... never contains two elements.
  - ... only contains elements if the linear equation corresponds to a constant function.
  - ... cannot be the empty set.
- b) If a linear equation has exactly one solution ...
  - ... the graph of the corresponding linear function intersects the x-axis.
  - ... the equation does not contain any parameters.
  - ... the solution must be an integer.
  - ... no other linear equation can have the same solution.
- c) If a linear equation has the solution  $x = 2$ , it can be concluded that ...
  - ...  $x = 3$  is not a solution.
  - ... the graph of the corresponding linear function intersects the x-axis at  $x = 2$ .
  - ...  $P(2|0)$  is a point of the graph of the corresponding linear function.
  - ...  $P(0|2)$  is a point of the graph of the corresponding linear function.