Exercises 3   Function
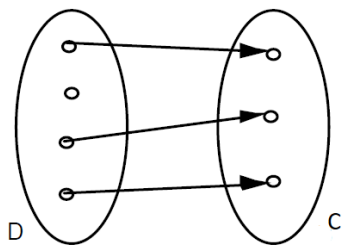### Domain, codomain, range, graph

Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
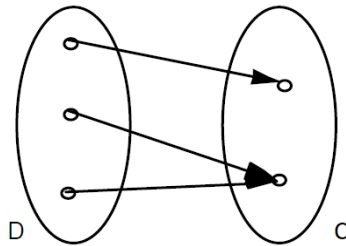- be able to determine values of a given function.

Problems

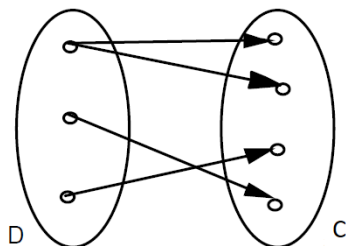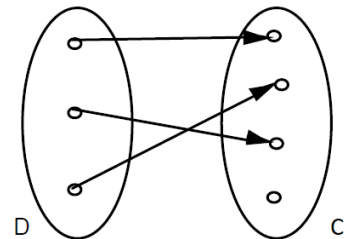3.1   Which of the following relations are functions? Explain your answer.

a)

b)

c)

d)

e)   D = set of all courses in the FHGR Tourism bachelor programme
       C = set of all FHGR lecturers
       f: D → C, c ↦ l = f(c) = lecturer of course c

f)   D = {1992, 1993, ... , 2001, 2002}
            C = set of all human beings aged between 20 and 30
            f: $D \to C$, $y \mapsto p = f(y)$ = person who was born in the year y

g)   D = set of all human beings aged between 20 and 30
            C = {1992, 1993, ... , 2001, 2002}
            f: $D \to C$, $p \mapsto y = f(p)$ = year of birth of person p

h)   f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto y = f(x) = x^2$

i)   f: $\mathbb{R}^+ \to \mathbb{R}$, $x \mapsto y = f(x)$ = number whose square is x

        Notice:
        - $\mathbb{R}^+$ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.

j)   f: $\mathbb{R} \to \mathbb{R}$, $t \mapsto b = f(t)$ = bank account balance at time t

3.2   Determine the range R of the functions below:

a)   D = {January, February, March, ..., December}
            C = {A, B, C, ..., Z}
            f: $D \to C$, $m \mapsto l = f(m)$ = initial letter of month m

b)   D = set of all neighbouring countries of Switzerland
            C = set of all European cities
            c: $D \to C$, $x \mapsto y = c(x)$ = capital of neighbouring country x

c)   function f in problem 3.1 g)

d)   function f in problem 3.1 h)

3.3   a)   f: $\mathbb{R} \to \mathbb{R}$, $x \mapsto f(x) = x^3 - x$

            Determine the following values:

        i)   f(1)   ii)   f(-2)   iii)   f(a)

        iv) $f(b^2)$ v)   f(a - b)   vi) $f(x^3 - x)$

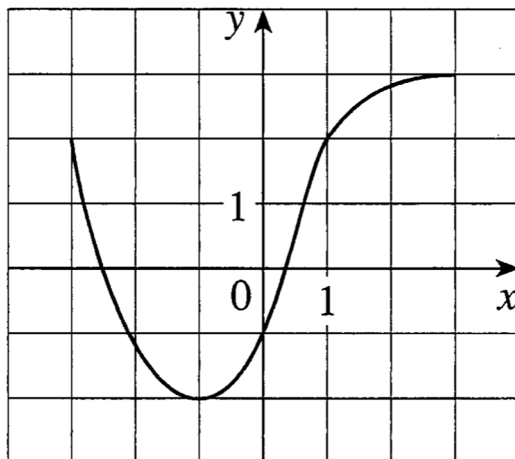   b)   g: $\mathbb{R} \setminus \{-1\} \to \mathbb{R}$, $x \mapsto g(x) = \dfrac{x^2}{x+1}$

            Determine the following values:

        i)   g(2)   ii)   g(-3)   iii)   g(a)

        iv) $g(b^2)$ v)   g(a - b)   vi) $g\left(\dfrac{x^2}{x+1}\right)$

3.4   (see next page)

3.4   The graph of a function f ist given as follows:

a)   State the value of f(-1).

b)   Estimate the value of f(2).

c)   For what values of x is f(x) = 2?

d)   Estimate the values of x such that f(x) = 0.

e)   State the domain D of f.

f)   State the range R of f.

3.5   Decide which statements are true or false. Put a mark into the corresponding box.
       In each problem a) to c), exactly one statement is true.

a)   The range of the function f: $\{x: x \in \mathbb{R} \text{ and } x \geq 4\} \rightarrow \mathbb{R}, x \mapsto y = f(x) = \sqrt{x - 4}$, is the set ...

       ☐   ... $\{x: x \in \mathbb{R} \text{ and } x \geq 4\}$
       ☐   ... $\{y: y \in \mathbb{R} \text{ and } y \geq 4\}$
       ☐   ... $\mathbb{R}$
       ☐   ... $\mathbb{R}_0^+$

b)   f cannot be a function if ...

       ☐   ... the domain of f is no number set.
       ☐   ... the codomain of f contains more elements than the domain of f.
       ☐   ... the domain of f contains more elements than the codomain of f.
       ☐   ... at least one element of the domain of f has more than one image.

c)   If the range of a function contains as many elements as the domain, it can be concluded that …

       ☐   ... the range is the same set as the domain.
       ☐   ... the codomain contains as many elements as the domain.
       ☐   ... each element of the codomain is also an element of the range.
       ☐   ... no element of the range is associated to more than one element of the domain.