Exercises 3 Function
Domain, codomain, range, graph

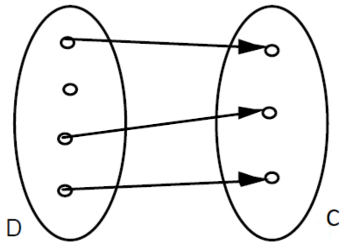
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

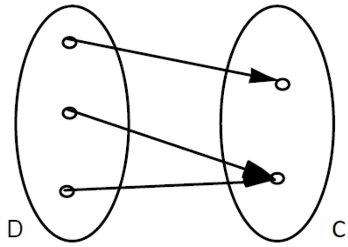
Problems

3.1 Which of the following relations are functions? Explain your answer.

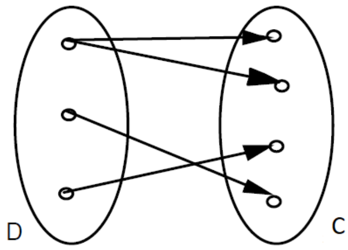
a)



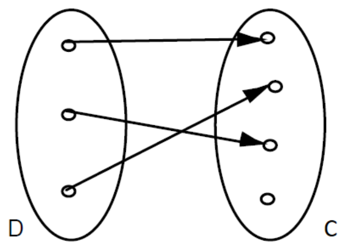
b)



c)



d)



e) D = set of all courses in the FHGR Tourism bachelor programme
C = set of all FHGR lecturers
f: $D \rightarrow C$, $c \mapsto l = f(c)$ = lecturer of course c

f) $D = \{1992, 1993, ..., 2001, 2002\}$
C = set of all human beings aged between 20 and 30
$f: D \to C, y \mapsto p = f(y)$ = person who was born in the year y

g) D = set of all human beings aged between 20 and 30
$C = \{1992, 1993, ..., 2001, 2002\}$
$f: D \to C, p \mapsto y = f(p)$ = year of birth of person p

h) $f: \mathbb{R} \to \mathbb{R}, x \mapsto y = f(x) = x^2$

i) $f: \mathbb{R}^+ \to \mathbb{R}, x \mapsto y = f(x)$ = number whose square is x

Notice:
- $\mathbb{R}^+$ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.

j) $f: \mathbb{R} \to \mathbb{R}, t \mapsto b = f(t)$ = bank account balance at time t

3.2 Determine the range R of the functions below:

a) D = {January, February, March, ..., December}
$C = \{A, B, C, ..., Z\}$
$f: D \to C, m \mapsto l = f(m)$ = initial letter of month m

b) D = set of all neighbouring countries of Switzerland
C = set of all European cities
$c: D \to C, x \mapsto y = c(x)$ = capital of neighbouring country x

c) function f in problem 3.1 g)

d) function f in problem 3.1 h)

3.3 a) $f: \mathbb{R} \to \mathbb{R}, x \mapsto f(x) = x^3 - x$

Determine the following values:

i)	$f(1)$	ii)	$f(-2)$	iii)	$f(a)$
iv)	$f(b^2)$	v)	$f(a - b)$	vi)	$f(x^3 - x)$

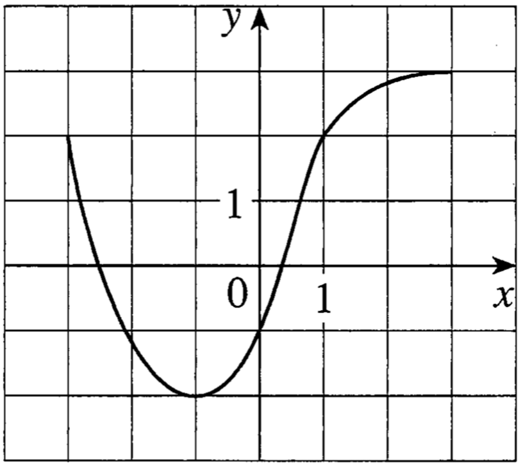
b) $g: \mathbb{R} \setminus \{-1\} \to \mathbb{R}, x \mapsto g(x) = \frac{x^2}{x+1}$

Determine the following values:

i)	$g(2)$	ii)	$g(-3)$	iii)	$g(a)$
iv)	$g(b^2)$	v)	$g(a - b)$	vi)	$g\left(\frac{x^2}{x+1}\right)$

3.4 (see next page)

3.4 The graph of a function f ist given as follows:



a) State the value of f(-1).

b) Estimate the value of f(2).

c) For what values of x is f(x) = 2?

d) Estimate the values of x such that f(x) = 0.

e) State the domain D of f.

f) State the range R of f.

3.5 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The range of the function f: $\{x: x \in \mathbb{R} \text{ and } x \geq 4\} \rightarrow \mathbb{R}$, $x \mapsto y = f(x) = \sqrt{x - 4}$, is the set ...

- ☐ ... $\{x: x \in \mathbb{R} \text{ and } x \geq 4\}$
- ☐ ... $\{y: y \in \mathbb{R} \text{ and } y \geq 4\}$
- ☐ ... $\mathbb{R}$
- ☐ ... $\mathbb{R}_0^+$

b) f cannot be a function if ...

- ☐ ... the domain of f is no number set.
- ☐ ... the codomain of f contains more elements than the domain of f.
- ☐ ... the domain of f contains more elements than the codomain of f.
- ☐ ... at least one element of the domain of f has more than one image.

c) If the range of a function contains as many elements as the domain, it can be concluded that …

- ☐ ... the range is the same set as the domain.
- ☐ ... the codomain contains as many elements as the domain.
- ☐ ... each element of the codomain is also an element of the range.
- ☐ ... no element of the range is associated to more than one element of the domain.

Answers

3.1 a) no function
No element (instead of exactly one element) of C is associated to one of the elements of D.

b) function

c) no function
Two elements (instead of exactly one element) of C are associated to one of the elements of D.

d) function

e) no function
More than one element (instead of exactly one element) of C are associated to some elements of D.

f) no function
Many elements (instead of exactly one element) of C are associated to each element of D.

g) function

h) function

i) no function
Two elements (instead of exactly one element) of $\mathbb{R}$ are associated to each element of $\mathbb{R}^+$.

j) function

3.2 a) R = {A, D, F, J, M, N, O, S}

b) R = {Berlin, Vienna, Vaduz, Rome, Paris}

c) R = C

d) $R = \mathbb{R}_0^+$

Notice:
- $\mathbb{R}_0^+$ is the set of all positive real numbers, including zero, i.e. $\mathbb{R}_0^+ = \{x: x \in \mathbb{R} \text{ and } x \geq 0\}$.

3.3 a)
i) $f(1) = 1^3 - 1 = 0$
ii) $f(-2) = (-2)^3 - (-2) = -6$
iii) $f(a) = a^3 - a$
iv) $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$
v) $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$
vi) $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$

b)
i) $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$
ii) $g(-3) = \frac{(-3)^2}{-3+1} = -\frac{9}{2}$
iii) $g(a) = \frac{a^2}{a+1}$
iv) $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$
v) $g(a - b) = \frac{(a-b)^2}{(a-b)+1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$
vi) $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right)+1} = \frac{x^4}{x^3 + 2x^2 + 2x + 1}$

3.4 a) $f(-1) = -2$

b) $f(2) \approx 2.8$

c) $x_1 = -3$, $x_2 = 1$

d) $x_1 \approx -2.5$, $x_2 \approx 0.3$

e) $D = \{x: x \in \mathbb{R} \text{ and } -3 \le x \le 3\} = [-3,3]$

f) $R = \{y: y \in \mathbb{R} \text{ and } -2 \le y \le 3\} = [-2,3]$

3.5 a) 4^{th} statement

b) 4^{th} statement

c) 4^{th} statement