Indefinite integral

Ex.: Financial mathematics

Given the marginal cost function C' for the production of a commodity:

$C'(x) = 3x + 50$

What is the cost function C?

$C(x) = \dots ?$

General problem

Given a function f. What function F is such that F' = f ?

Ex.: $f(x) = 2x$

$\Rightarrow$ $F_1(x) = x^2$ as $F_1'(x) = 2x = f(x)$
$F_2(x) = x^2 + 1$ as $F_2'(x) = 2x + 0 = 2x = f(x)$
$F_3(x) = x^2 - 4$ as $F_3'(x) = 2x + 0 = 2x = f(x)$
…
$F(x) = x^2 + C \ (C \in \mathbb{R})$ as $F'(x) = 2x + 0 = 2x = f(x)$

These are already all functions F with F' = f. There are no additional functions F with equations different from $F(x) = x^2 + C \ (C \in \mathbb{R})$.

$f(x) = 8x^3$

$\Rightarrow$ $F_1(x) = 2x^4$ as $F_1'(x) = 8x^3 = f(x)$
$F_2(x) = 2x^4 + 5$ as $F_2'(x) = 8x^3 + 0 = 8x^3 = f(x)$
$F_3(x) = 2x^4 - 11$ as $F_3'(x) = 8x^3 + 0 = 8x^3 = f(x)$
…
$F(x) = 2x^4 + C \ (C \in \mathbb{R})$ as $F'(x) = 8x^3 + 0 = 8x^3 = f(x)$

Definitions

F is called an **antiderivative** of f if its derivative F' is equal to f, i.e. $F'(x) = f(x)$.

The set of all antiderivatives of the function f is called the **indefinite integral** of f, denoted $\int f(x)\,dx$.

Ex.: $f(x) = 8x^3$

All antiderivatives F have the form $F(x) = 2x^4 + C \ (C \in \mathbb{R})$.
Therefore, we write $\int f(x)\,dx = \int 8x^3\,dx = 2x^4 + C$

$f(x) = 12x^2$

$\int f(x)\,dx = \int 12x^2\,dx = 4x^3 + C$

$\int 2x\,dx = x^2 + C$

$\int 3\,e^{3x}\,dx = e^{3x} + C$

$C \ (C \in \mathbb{R})$ is called the **integration constant**.