

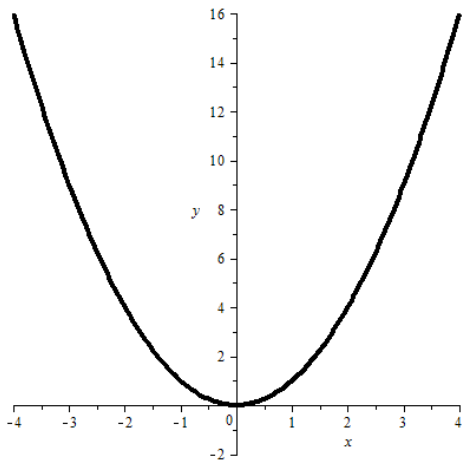
Quadratic function

Definition

$f: D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \mapsto y = f(x) = ax^2 + bx + c$ general form	$(a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}, c \in \mathbb{R})$
$y = f(x) = a(x - u)^2 + v$ vertex form	$(a \in \mathbb{R} \setminus \{0\}, u \in \mathbb{R}, v \in \mathbb{R})$

Graph

1. $y = f(x) = x^2$ ($a = 1, u = 0, v = 0$)

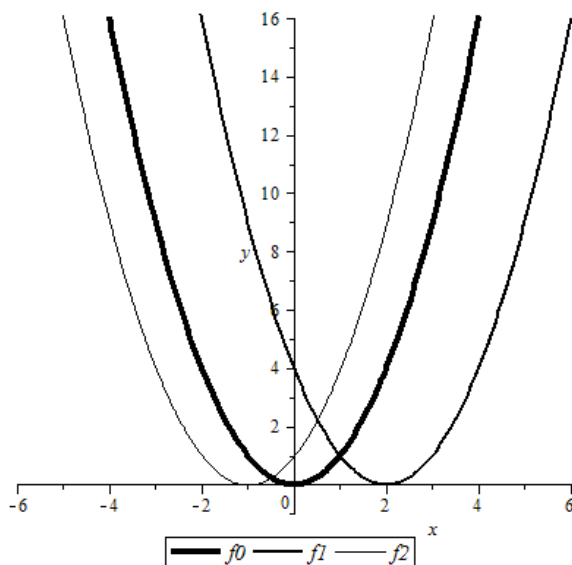


2. Parameter **u** (**varying u**, keeping a and v constant)

$$y = f_0(x) = x^2 \quad (a = 1, u = 0, v = 0)$$

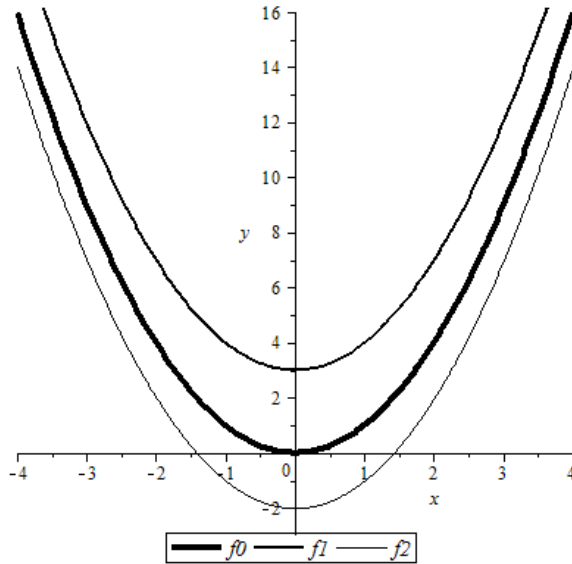
$$y = f_1(x) = (x - 2)^2 \quad (a = 1, u = 2, v = 0)$$

$$y = f_2(x) = (x + 1)^2 \quad (a = 1, u = -1, v = 0)$$



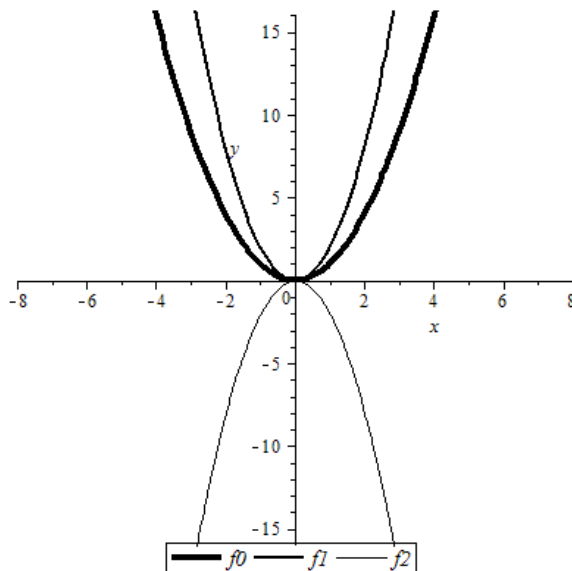
3. Parameter **v** (varying **v**, keeping **a** and **u** constant)

$$\begin{aligned} y = f_0(x) &= x^2 & (a = 1, u = 0, v = 0) \\ y = f_1(x) &= x^2 + 3 & (a = 1, u = 0, v = 3) \\ y = f_2(x) &= x^2 - 2 & (a = 1, u = 0, v = -2) \end{aligned}$$



4. Parameter **a** (varying **a**, keeping **u** and **v** constant)

$$\begin{aligned} y = f_0(x) &= x^2 & (a = 1, u = 0, v = 0) \\ y = f_1(x) &= 2x^2 & (a = 2, u = 0, v = 0) \\ y = f_2(x) &= -2x^2 & (a = -2, u = 0, v = 0) \end{aligned}$$

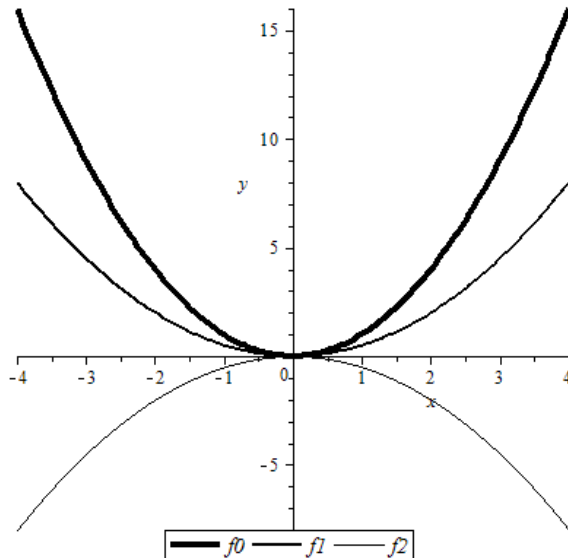


5. Parameter **a** (varying **a**, keeping **u** and **v** constant)

$$y = f_0(x) = x^2 \quad (a = 1, u = 0, v = 0)$$

$$y = f_1(x) = \frac{1}{2}x^2 \quad (a = \frac{1}{2}, u = 0, v = 0)$$

$$y = f_2(x) = -\frac{1}{2}x^2 \quad (a = -\frac{1}{2}, u = 0, v = 0)$$



6. The **graph** of a quadratic function is a **parabola**.

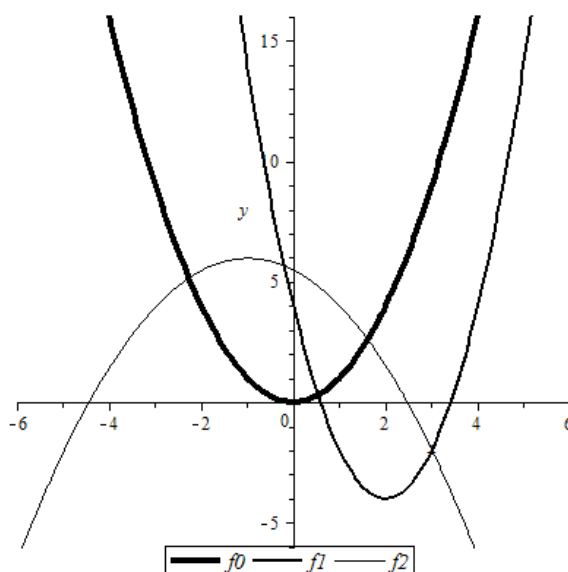
The parameter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** **V** of the parabola: $V(u|v)$

$$y = f_0(x) = x^2 \quad (a = 1, u = 0, v = 0) \quad V(0|0)$$

$$y = f_1(x) = 2(x - 2)^2 - 4 \quad (a = 2, u = 2, v = -4) \quad V(2|-4)$$

$$y = f_2(x) = -\frac{1}{2}(x + 1)^2 + 6 \quad (a = -\frac{1}{2}, u = -1, v = 6) \quad V(-1|6)$$

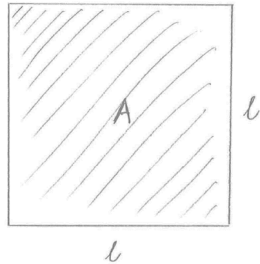


Examples

1. Nature/Physics: Trajectories of water in a fountain



2. Geometry: Square

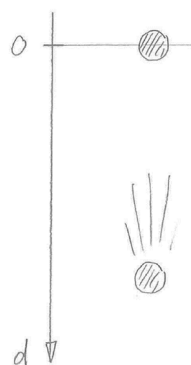


Area A for side length l : $A = l^2$

$f: \mathbb{R}^+ \rightarrow \mathbb{R}$

$l \mapsto A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t : $d = \frac{1}{2}gt^2$ (g = gravity field strength)

$f: \mathbb{R} \rightarrow \mathbb{R}$

$t \mapsto d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand