## Function

## Definition and examples

Def.: A function f is a rule that associates to each element x in a set D exactly one element y in a set C .


The function f maps the set D onto the set C .
f: $\quad \mathrm{D} \rightarrow \mathrm{C}$

$$
\mathrm{x} \mapsto \mathrm{y}=\mathrm{f}(\mathrm{x}) \quad \text { ("f of } \mathrm{x} \text { ") }
$$

The set $D$ is the domain, the set $C$ is the codomain, and the set $R$ is the range of the function $f$.
The element y is the image of the element x .
or (if D and C are number sets): y is the value of f at x .


Ex.: 1. $\quad \mathrm{D}=$ set of all Swiss holiday resorts $\mathrm{C}=$ set of all human beings
f: $\quad \mathrm{D} \rightarrow \mathrm{C}$
$\mathrm{r} \mapsto \mathrm{d}=\mathrm{f}(\mathrm{r})=$ director of holiday resort r

2. $\mathrm{D}=$ set of all countries of the world

C $=$ set of all cities of the world

$$
\begin{array}{ll}
\text { f: } \quad & \mathrm{D} \rightarrow \mathrm{C} \\
\mathrm{a} \rightarrow \mathrm{~b}=\mathrm{f}(\mathrm{a})=\text { capital of country a }
\end{array}
$$

3. Cable car company
$\mathrm{D}=\mathbb{N} \quad$ (= set of natural numbers)
$C=\mathbb{R} \quad$ (= set of real numbers)
f: $\quad \mathbb{N} \rightarrow \mathbb{R}$
$\mathrm{n} \mapsto \mathrm{r}=\mathrm{f}(\mathrm{n})=$ revenue (e.g. in CHF) when n tickets are sold
4. $\mathrm{D}=\mathbb{N}$
$\mathrm{C}=\mathbb{Z}$
f: $\quad \mathbb{N} \rightarrow \mathbb{Z}$
$\mathrm{n} \mapsto \mathrm{y}=\mathrm{f}(\mathrm{n})=\mathrm{n}-4$

5. $\mathrm{D}=\mathrm{C}=\mathbb{R}$

$$
\begin{array}{ll}
\mathrm{p}: & \mathbb{R} \rightarrow \mathbb{R} \\
& \mathrm{x} \mapsto \mathrm{y}=\mathrm{p}(\mathrm{x})=\frac{\mathrm{x}^{3}-3}{2 \mathrm{x}^{2}+1}
\end{array}
$$

## Representation of a function

## Arrow diagram



## Table of values

| x | y |
| :---: | :---: |
| 1 | 1 |
| 3 | 9 |
| 5 | 25 |
| -5 | 25 |
| 0.4 | 0.16 |
| $\ldots$ | $\ldots$ |

## Equation

$$
\text { f: } \begin{aligned}
& \mathbb{R} \rightarrow \mathbb{R} \\
& x \not \mapsto y=f(x)=x^{2}
\end{aligned}
$$

## Graph



