Exercises 14 Differentiation rules
Coefficient, sum, product, exponential function, higher-order derivatives

Objectives

- be able to apply the coefficient, sum, and product rules to determine the derivative of a function.
- be able to determine a higher-order derivative of a function.

Problems

14.1 Determine the derivative by applying the **coefficient rule**:

a)	$f(x) = 3x^5$	b)	$f(x) = -4x^3$	c)	$f(x) = -x^{10}$
d)	$f(x) = a \cdot x^3$	e)	$f(x) = n \cdot x^{n-1}$	f)	$f(x) = 9 \cdot 3^x$
g)	$s(t) = \frac{1}{2} g \cdot t^2$	h)	$S(T) = \alpha \cdot T^4$	i)	$C(x) = (-3x)^3$

14.2 Determine the derivative by applying the **sum rule**:

a)	$f(x) = x^5 + x^6$	b)	$f(x) = x^{10} - x^9$	c)	$f(x) = 1 + x + 3x^3$
d)	$f(x) = \frac{1}{4} x^4 + 3x^2 - 2$	e)	$f(x) = 3x^2(x - 2)$	f)	$f(x) = - 3x^8 + x^5 - 3x + 99$
g)	$f(x) = ax^2 + bx + c$	h)	$f(x) = 3(a^2 - 2ax + x^2)$	i)	$f(x) = \frac{x^3}{3} - \frac{3}{x^3}$
j)	$s(t) = s_0 + v_0 t + \frac{1}{2} g \cdot t^2$	k)	$V(r) = - \frac{a}{r} + \frac{b}{r^2}$	l)	$C(n) = C_0(1 + nr)$

14.3 Determine the derivative by applying the **product rule**:

a)	$f(x) = x \cdot e^x$	b)	$f(x) = x^3 \cdot 3^x$
c)	$f(x) = - 2x^5(x - 1)$	d)	$f(x) = (2x - 1) \cdot e^x$
e)	$f(x) = (2x - 1)(- 3x^2 - x + 1)$	f)	$V(r) = e^r \left(a \cdot r^2 - \frac{b}{r^3}\right)$

14.4 Determine the derivative of the exponential functions below:

a)	$f(x) = e^{4x}$	b)	$f(x) = e^{-x}$
c)	$f(x) = e^{-x^2}$	d)	$f(x) = e^{x^2-2x+5}$

14.5 Determine the derivative by applying the appropriate differentiation rule(s), and simplify the expression as far as possible:

a)	$f(x) = (x - 2)\, e^{2x}$	b)	$f(x) = (2 - x^2)\, e^{-x}$
c)	$f(x) = (3x^3 - 2x^2 + x - 1)\, e^{-2x}$	d)	$P(v) = av^2\, e^{-bv^2}$

14.6 Determine the derivative (rate of change) of the functions below at the indicated position:

a)	f in 14.1 b)	$x = 2$	b)	s in 14.1 g)	$t = 4$
c)	f in 14.2 g)	$x = -1$	d)	P in 14.5 d)	$v = 1$

14.7 (see next page)

14.7 Determine the second and third derivatives of the functions in problem …

a) … 14.1 a)
b) … 14.2 g)
c) … 14.3 a)
d) … 14.4 c)

14.8 Determine the indicated higher-order derivatives:

a) f ''(-1) with function f in 14.1 a)

Hint:
- You have already determined f ''(x) in 14.7 a).

b) f '''(2) with function f in 14.4 c)

Hint:
- You have already determined f '''(x) in 14.7 d).

14.9 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The third derivative of a function is a ...

- ☐ ... constant function if the second derivative is a quadratic function.
- ☐ ... quadratic function if the second derivative is a linear function.
- ☐ ... linear function if the first derivative is a quadratic function.
- ☐ ... constant function if the first derivative is a quadratic function.

b) The derivative of a ...

- ☐ ... product is the product of the derivatives of the single factors.
- ☐ ... product is the sum of the derivatives of the single factors.
- ☐ ... sum is the sum of the derivatives of the single addends.
- ☐ ... constant is the constant itself.

c) If f(x) = c·g(x)·h(x) then f '(x) = ...

- ☐ ... 0
- ☐ ... c·g'(x)·h'(x)
- ☐ ... c·g(x)·h'(x) + c·g'(x)·h(x)
- ☐ ... c·g'(x)·h'(x) + c·g(x)·h(x)

Answers

14.1 a) $f'(x) = 3 \cdot 5x^4 = 15x^4$

b) $f'(x) = (-4)\ 3x^2 = -12x^2$

c) $f'(x) = (-1)\ 10x^9 = -10x^9$

d) $f'(x) = a \cdot 3x^2 = 3ax^2$

Hint:
- a is a constant.

e) $f'(x) = n(n-1)x^{n-2}$

f) $f'(x) = 9 \cdot 3^x \cdot \ln(3)$

g) $s'(t) = \frac{g}{2}\ 2t = gt$

Hints:
- The name of the function is s, and the variable is t.
- g is a constant.

h) $S'(T) = \alpha \cdot 4T^3 = 4\alpha T^3$

i) $C'(x) = -\ 81x^2$

14.2 a) $f'(x) = 5x^4 + 6x^5$ b) $f'(x) = 10x^9 - 9x^8$ c) $f'(x) = 1 + 9x^2$

d) $f'(x) = x^3 + 6x$ e) $f'(x) = 9x^2 – 12x$ f) $f'(x) = -\ 24x^7 + 5x^4 - 3$

g) $f'(x) = 2ax + b$ h) $f'(x) = -\ 6a + 6x$ i) $f'(x) = x^2 + \frac{9}{x^4}$

j) $s'(t) = v_0 + gt$ k) $V'(r) = \frac{a}{r^2} - \frac{2b}{r^3}$ l) $C'(n) = C_0 \cdot r$

14.3 a) $f'(x) = e^x + x \cdot e^x$

b) $f'(x) = 3x^2 \cdot 3^x + x^3 \cdot 3^x \cdot \ln(3)$

c) $f'(x) = -\ 2(5x^4(x - 1) + x^5)$

d) $f'(x) = 2 \cdot e^x + (2x - 1) \cdot e^x$

e) $f'(x) = 2(-\ 3x^2 - x + 1) + (2x - 1)\ (-\ 6x - 1)$

f) $V'(r) = e^r \left(a \cdot r^2 - \frac{b}{r^3}\right) + e^r \left(2a \cdot r + \frac{3b}{r^4}\right)$

Hints:
- V is the name of the function, and r is the variable.
- a and b are constants.

14.4 a) $f'(x) = 4\ e^{4x}$ b) $f'(x) = (-1)\ e^{-x} = -\ e^{-x}$

c) $f'(x) = -\ 2x \cdot e^{-x^2}$ d) $f'(x) = (2x - 2)\ e^{x^2 - 2x + 5}$

14.5 a) $f'(x) = e^{2x} + (x - 2)\ 2\ e^{2x} = (2x - 3)\ e^{2x}$

b) $f'(x) = -\ 2x\ e^{-x} + (2 - x^2)\ (-1)\ e^{-x} = (x^2 - 2x - 2)\ e^{-x}$

c) $f'(x) = (9x^2 - 4x + 1)\ e^{-2x} + (3x^3 - 2x^2 + x - 1)\ (-2)\ e^{-2x} = (-\ 6x^3 + 13x^2 - 6x + 3)\ e^{-2x}$

d) $P'(v) = a\left(2v\ e^{-bv^2} + v^2(-2bv)\ e^{-bv^2}\right) = 2av\ (1 - bv^2)\ e^{-bv^2}$

14.6 (see next page)

14.6 a) $f'(2) = -48$

b) $s'(4) = 4g$

c) $f'(-1) = -2a + b$

d) $P'(1) = 2a\,(1 - b)\,e^{-b}$

14.7 a) 14.1 a)
$f''(x) = 15 \cdot 4x^3 = 60x^3$
$f'''(x) = 60 \cdot 3x^2 = 180x^2$

b) 14.2 g)
$f''(x) = 2a \cdot 1 = 2a$
$f'''(x) = 0$

c) 14.3 a)
$f''(x) = e^x + (e^x + x \cdot e^x) = (x + 2)\,e^x$
$f'''(x) = e^x + (x + 2)\,e^x = (x + 3)\,e^x$

d) 14.4 c)
$f''(x) = -2\left(e^{-x^2} + x\,(-2x)\,e^{-x^2}\right) = 2\,(2x^2 - 1)\,e^{-x^2}$
$f'''(x) = 2\left(4x\,e^{-x^2} + (2x^2 - 1)(-2x)\,e^{-x^2}\right) = 4x\,(-2x^2 + 3)\,e^{-x^2}$

14.8 a) $f''(-1) = 60\,(-1)^3 = -60$

b) $f'''(2) = 4 \cdot 2\,(-2 \cdot 2^2 + 3)\,e^{-2^2} = -\frac{40}{e^4}$

14.9 a) 4^{th} statement

b) 3^{rd} statement

c) 3^{rd} statement