Exercises 11 Exponential function and equations Compound interest, nominal/effective annual interest rate

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded more than once per year.
- know and understand what a nominal and an effective annual interest rate is.
- be able to treat specific compound interest tasks.

Problems

11.1 An initial capital C_0 = 1000 CHF is invested at a nominal annual interest rate r_a = 10%, compounded ...

a) ... quarterly.

i) Determine the quarterly interest rate r.

ii) Determine the capitals after one, two, and three years.

iii) Determine the effective annual interest rate r_a^*.

b) ... monthly.

i) Determine the monthly interest rate r.

ii) Determine the capitals after one, two, and three years.

iii) Determine the effective annual interest rate r_a^*.

11.2 Determine the effective annual interest rate for a nominal annual interest rate of 6%, compounded ...

a) ... annually.

b) ... semiannually.

c) ... quarterly.

d) ... monthly.

e) ... daily (1 year = 360 days).

11.3 What is the future value if 3200 CHF is invested for 5 years at 8% compounded quarterly?

11.4 Find the interest that will be earned if 10'000 CHF is invested for 3 years at 9% compounded monthly.

11.5 The formula

$$C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n$$

is used for calculating the future capital C_n in a compound interest scheme.

Solve the formula for C_0, r_a, and n.

11.6 What amount of money do parents need to deposit in an account earning 10%, compounded monthly, so that it will grow to 40'000 CHF for their son's college tuition in 18 years?

11.7 (see next page)

11.7 An initial capital of 1000 CHF amounts to 1500 CHF if it is invested for 10 years at an unknown annual interest rate, compounded quarterly.

Determine the ...

a) ... nominal annual interest rate.

b) ... effective annual interest rate.

11.8 How long (in months) would a capital have to be invested at 6%, compounded monthly, to double its value?

11.9 Ms Good wants to invest 100'000 CHF. Her bank makes two offers:

A effective annual interest rate of 8.5%

B nominal annual interest rate of 8%, compounded monthly

Which offer is better, offer A or offer B?

11.10 How long (in years) would 1000 CHF have to be invested at 2.5%, compounded daily, to earn 250 CHF interest?

11.11 At what nominal annual interest rate, compounded quarterly, would 20'000 CHF have to be invested to amount to 26'400 CHF in 7 years?

11.12 A couple needs 150'000 CHF as a down payment for a home. If they invest the 100'000 CHF they have at 8% compounded quarterly, how long will it take for the money to grow into 150'000 CHF?

11.13 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The nominal annual interest rate ...

- ☐ ... is generally higher than the effective annual interest rate.
- ☐ ... is equal to the effective annual interest rate if interest is compounded annually.
- ☐ ... is half as much as the effective annual interest rate if interest is compounded semiannually.
- ☐ ... depends on the compounding period.

b) In a compound interest scheme where interest is compounded m (m > 1) times per year ...

- ☐ ... the growth factor is m times as high as if interest is compounded only once per year.
- ☐ ... the effective annual interest rate is m times lower than if interest is compounded only once per year.
- ☐ ... the capital grows faster than if interest is compounded only once per year.
- ☐ ... the capital grows more slowly than if interest is compounded only once per year.

c) If an initial capital of 1000 CHF grows to 1100 CHF in one year and interest is compounded semiannually ...

- ☐ ... the effective annual interest rate is less than 10%.
- ☐ ... the effective annual interest rate is greater than 10%.
- ☐ ... the nominal annual interest rate is less than 10%.
- ☐ ... the nominal annual interest rate is greater than 10%.

Answers

11.1 a) i) $r = \frac{r_a}{m} = \frac{10\%}{4} = 2.5\%$

ii) $C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n$ where m = 4, n = number of quarters

after 1 year: n = 4
$C_4 = 1000 \left(1 + \frac{10\%}{4}\right)^4$ CHF = 1103.81 CHF (rounded)

after 2 years: n = 8
$C_8 = 1000 \left(1 + \frac{10\%}{4}\right)^8$ CHF = 1218.40 CHF (rounded)

after 3 years: n = 12
$C_{12} = 1000 \left(1 + \frac{10\%}{4}\right)^{12}$ CHF = 1344.89 CHF (rounded)

iii) $r_a^* = \left(1 + \frac{r_a}{m}\right)^m - 1 = \left(1 + \frac{10\%}{4}\right)^4 - 1 = 0.1038 = 10.38\%$ (rounded)

b) i) $r = \frac{r_a}{m} = \frac{10\%}{12} = 0.83\%$ (rounded)

ii) $C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n$ where m = 12, n = number of months

after 1 year: n = 12
$C_{12} = 1000 \left(1 + \frac{10\%}{12}\right)^{12}$ CHF = 1104.71 CHF (rounded)

after 2 years: n = 24
$C_{24} = 1000 \left(1 + \frac{10\%}{12}\right)^{24}$ CHF = 1220.39 CHF (rounded)

after 3 years: n = 36
$C_{36} = 1000 \left(1 + \frac{10\%}{12}\right)^{36}$ CHF = 1348.18 CHF (rounded)

iii) $r_a^* = \left(1 + \frac{r_a}{m}\right)^m - 1 = \left(1 + \frac{10\%}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$ (rounded)

11.2 $r_a^* = \left(1 + \frac{r_a}{m}\right)^m - 1$ $r_a = 6\%$

a) m = 1 $r_a^* = 6\%$

b) m = 2 $r_a^* = 6.09\%$

c) m = 4 $r_a^* = 6.136\%$ (rounded)

d) m = 12 $r_a^* = 6.168\%$ (rounded)

e) m = 360 $r_a^* = 6.183\%$ (rounded)

11.3 $C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n$ where C_0 = 3200 CHF, r_a = 8%, m = 4, n = 5·4 = 20

$\Rightarrow$ C_{20} = 4755.03 CHF (rounded)

11.4 Interest = $C_n - C_0$
$C_n = C_0 \left(1 + \frac{r_a}{m}\right)^n$ where C_0 = 10'000 CHF, r_a = 9%, m = 12, n = 3·12 = 36

$\Rightarrow$ $C_{36} - C_0$ = 3086.45 CHF (rounded)

11.5 see formulary

11.6 $C_0 = \frac{C_n}{\left(1+\frac{r_a}{m}\right)^n}$ where C_n = 40'000 CHF, r_a = 10%, m = 12, n = 18·12 = 216

$\Rightarrow$ C_0 = 6661.46 CHF (rounded)

11.7 a) $r_a = m\left(\sqrt[n]{\frac{C_n}{C_0}} - 1\right)$ where C_0 = 1000 CHF, C_n = 1500 CHF, m = 4, n = 10·4 = 40

$\Rightarrow$ r_a = 4.08% (rounded)

b) $r_a^* = \left(1+\frac{r_a}{m}\right)^m - 1$

$\Rightarrow$ r_a^* = 4.14% (rounded)

11.8 $n = \frac{\lg\left(\frac{C_n}{C_0}\right)}{\lg\left(1+\frac{r_a}{m}\right)}$ where $\frac{C_n}{C_0}$ = 2, r_a = 6%, m = 12

$\Rightarrow$ n = 138.98... $\rightarrow$ 139 months

11.9 A $r_a^*(A)$ = 8.5%

B $r_a^*(B) = \left(1+\frac{r_a}{m}\right)^m - 1$ where r_a = 8%, m = 12

$\Rightarrow$ $r_a^*(B)$ = 8.3%

$\Rightarrow$ $r_a^*(A) > r_a^*(B)$, i.e. offer A is better than offer B

11.10 $n = \frac{\lg\left(\frac{C_n}{C_0}\right)}{\lg\left(1+\frac{r_a}{m}\right)}$ where C_0 = 1000 CHF, C_n = 1250 CHF, r_a = 2.5%, m = 360

$\Rightarrow$ n = 3213.38… $\rightarrow$ 3214 days = 8.92… years $\rightarrow$ 9 years

11.11 $r_a = m\left(\sqrt[n]{\frac{C_n}{C_0}} - 1\right)$ where C_0 = 20'000 CHF, C_n = 26'400 CHF, m = 4, n = 7·4 = 28

$\Rightarrow$ r_a = 4.0% (rounded)

11.12 $n = \frac{\lg\left(\frac{C_n}{C_0}\right)}{\lg\left(1+\frac{r_a}{m}\right)}$ where C_0 = 100'000 CHF, C_n = 150'000 CHF, r_a = 8%, m = 4

$\Rightarrow$ n = 20.47... $\rightarrow$ 21 quarters = 5 years 3 months

11.13 a) 2nd statement

b) 3rd statement

c) 3rd statement