

Exercises 5 Linear function and equations Linear equations

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to treat applied tasks in economics by means of linear equations.

Problems

5.1 Determine the solution sets of the following equations:

- a) $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

5.2 Determine the solution sets of the following equations:

- a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b) $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c) $(x + 3)(x - 5) = (x - 3)^2$
- d) $(x - 5)(x - 2) = (x - 4)(x - 3)$
- e) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

5.3 Determine the solution sets of the following equations:

- a) $\frac{x+3}{5} = \frac{2x-8}{3}$
- b) $\frac{x+3}{4} + \frac{1-3x}{7} = 0$
- c) $\frac{2}{x-1} = \frac{1}{x-2}$
- d) $\frac{x}{x-1} = \frac{x-1}{x-2}$

5.4 The equations below are equations in the variable x . Furthermore, the equations contain parameters a and b . Therefore, the solution sets of the equations depend on the values of those parameters.

Solve the equations for x , and determine the solution sets.

Take into account that the parameters a and b can be any real numbers.

a) $x(a - 3) = a$

Hints:

- You may want to divide both sides of the equation by $a - 3$. However, this is not allowed if $a - 3 = 0$, i.e. if $a = 3$, as dividing by 0 is not defined.
- Solve the equation for the two cases $a \neq 3$ and $a = 3$.

b) $(x + 1)(b - 2) = 2bx$

c) $a(1 + x) - 1 = x(2a - 1)$

d) $(a - b)x = a$

5.5 The graph of a linear function f with slope a contains the point P . Find the equation of the linear function.

- a) $a = -5$ $P(5|-3)$
- b) $a = 2$ $P(3|0)$
- c) $a = 0$ $P(2|3)$

5.6 Alps Bikes uses the formula $B(t) = -400t + 5000$ to find the book value $B(t)$ (in CHF) of a mountain bike after the time t (in years) after its purchase.

- a) What do the numbers -400 and 5000 signify?
- b) How long will it take the mountain bike to depreciate completely?

5.7 Two items A and B depreciate linearly:

- Item A original value = 200 CHF
 depreciation = 16 CHF/year
- Item B original value = 240 CHF
 depreciation = 32 CHF/year

- a) How long will it take the two items to depreciate completely?
- b) Determine the point in time where both items have the same value.

Hint:

- Think of the temporal development of the values as linear functions.

5.8 Simple interest at an unknown rate is paid on an initial bank balance of 5000 CHF. The balance after five years is 5625 CHF.

- a) Determine the interest rate.
- b) How long would it take the balance to reach 7000 CHF?

5.9 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

- a) The solution set of a linear equation ...
 - ... always contains at least one element.
 - ... never contains two elements.
 - ... only contains elements if the linear equation corresponds to a constant function.
 - ... cannot be the empty set.
- b) If a linear equation has exactly one solution ...
 - ... the graph of the corresponding linear function intersects the x-axis.
 - ... the equation does not contain any parameters.
 - ... the solution must be an integer.
 - ... no other linear equation can have the same solution.
- c) If a linear equation has the solution $x = 2$, it can be concluded that ...
 - ... $x = 3$ is not a solution.
 - ... the graph of the corresponding linear function intersects the x-axis at $x = 2$.
 - ... $P(2|0)$ is a point of the graph of the corresponding linear function.
 - ... $P(0|2)$ is a point of the graph of the corresponding linear function.

Answers

- 5.1 a) $S = \{-1\}$
 b) $S = \{13\}$
 c) $S = \mathbb{R}$
 d) $S = \{ \}$

- 5.2 a) $S = \left\{ \frac{11}{2} \right\}$
 b) $S = \mathbb{R}$
 c) $S = \{6\}$
 d) $S = \{ \}$
 e) $S = \{0\}$

- 5.3 a) $S = \{7\}$
 b) $S = \{5\}$
 c) $S = \{3\}$
 d) $S = \{ \}$

- 5.4 a) if $a = 3$: no solution $\Rightarrow S = \{ \}$
 if $a \neq 3$: $x = \frac{a}{a-3}$ $\Rightarrow S = \left\{ \frac{a}{a-3} \right\}$
 b) if $b = -2$: no solution $\Rightarrow S = \{ \}$
 if $b \neq -2$: $x = \frac{b-2}{b+2}$ $\Rightarrow S = \left\{ \frac{b-2}{b+2} \right\}$
 c) if $a = 1$: $x \in \mathbb{R}$ $\Rightarrow S = \mathbb{R}$
 if $a \neq 1$: $x = 1$ $\Rightarrow S = \{1\}$
 d) if $a = b = 0$: $x \in \mathbb{R}$ $\Rightarrow S = \mathbb{R}$
 if $a = b \neq 0$: no solution $\Rightarrow S = \{ \}$
 if $a \neq b$: $x = \frac{a}{a-b}$ $\Rightarrow S = \left\{ \frac{a}{a-b} \right\}$

- 5.5 a) $y = f(x) = -5x + 22$

Hints:

- The equation of a linear function is $y = f(x) = ax + b$

- $P(5|-3)$ is a point of the graph of the linear function. Therefore, the coordinates of P must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$

- b) $y = f(x) = 2x - 6$

- c) $y = f(x) = 3$

- 5.6 a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year. The number 5000 indicates that the original value of the mountain bike was 5000 CHF.

- b) 12.5 years

Hint:

- Complete depreciation at time t means $B(t) = 0$.

- 5.7 a) item A: 12.5 years
item B: 7.5 years
b) $t = 2.5$ years (if the original value is meant to be at $t = 0$ years)
- 5.8 a) slope of the linear function: $a = 125$
interest rate $r = 2.5\%$
b) 16 years
- 5.9 a) 2nd statement
b) 1st statement
c) 3rd statement