

Exercises 3 **Function** **Domain, codomain, range, graph**

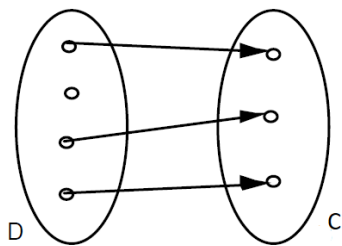
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

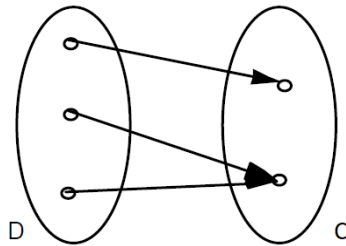
Problems

3.1 Which of the following relations are functions? Explain your answer.

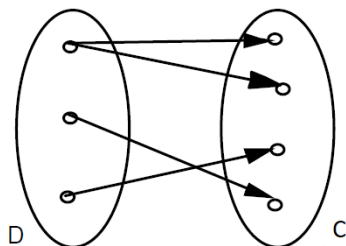
a)



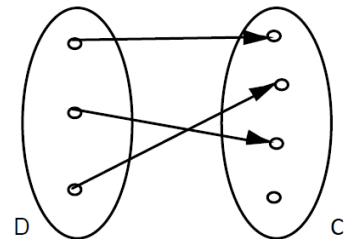
b)



c)



d)



- e) D = set of all courses in the FHGR Tourism bachelor programme
C = set of all FHGR lecturers
f: $D \rightarrow C, c \mapsto l = f(c) = \text{lecturer of course } c$

- f) $D = \{1991, 1992, \dots, 2000, 2001\}$
 $C = \text{set of all human beings aged between 20 and 30}$
 $f: D \rightarrow C, y \mapsto p = f(y) = \text{person who was born in the year } y$
- g) $D = \text{set of all human beings aged between 20 and 30}$
 $C = \{1991, 1992, \dots, 2000, 2001\}$
 $f: D \rightarrow C, p \mapsto y = f(p) = \text{year of birth of person } p$
- h) $f: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto y = f(x) = x^2$
- i) $f: \mathbb{R}^+ \rightarrow \mathbb{R}, x \mapsto y = f(x) = \text{number whose square is } x$
 Notice:
 - \mathbb{R}^+ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.
- j) $f: \mathbb{R} \rightarrow \mathbb{R}, t \mapsto b = f(t) = \text{bank account balance at time } t$

3.2 Determine the range R of the functions below:

- a) $D = \{\text{January, February, March, } \dots, \text{December}\}$
 $C = \{A, B, C, \dots, Z\}$
 $f: D \rightarrow C, m \mapsto l = f(m) = \text{initial letter of month } m$
- b) $D = \text{set of all neighbouring countries of Switzerland}$
 $C = \text{set of all European cities}$
 $c: D \rightarrow C, x \mapsto y = c(x) = \text{capital of neighbouring country } x$
- c) function f in problem 3.1 g)
- d) function f in problem 3.1 h)

3.3 a) $f: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto f(x) = x^3 - x$

Determine the following values:

- | | | |
|--------------|---------------|------------------|
| i) $f(1)$ | ii) $f(-2)$ | iii) $f(a)$ |
| iv) $f(b^2)$ | v) $f(a - b)$ | vi) $f(x^3 - x)$ |

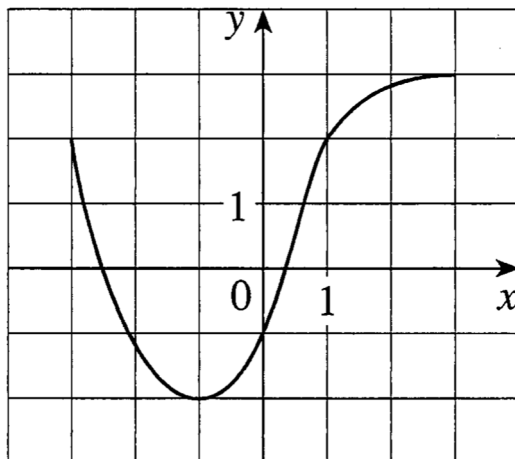
b) $g: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}, x \mapsto g(x) = \frac{x^2}{x+1}$

Determine the following values:

- | | | |
|--------------|---------------|-------------------------------------|
| i) $g(2)$ | ii) $g(-3)$ | iii) $g(a)$ |
| iv) $g(b^2)$ | v) $g(a - b)$ | vi) $g\left(\frac{x^2}{x+1}\right)$ |

3.4 (see next page)

3.4 The graph of a function f is given as follows:



- State the value of $f(-1)$.
- Estimate the value of $f(2)$.
- For what values of x is $f(x) = 2$?
- Estimate the values of x such that $f(x) = 0$.
- State the domain D of f .
- State the range R of f .

3.5 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

- The range of the function $f: \{x: x \in \mathbb{R} \text{ and } x \geq 4\} \rightarrow \mathbb{R}, x \mapsto y = f(x) = \sqrt{x - 4}$, is the set ...
 - ... $\{x: x \in \mathbb{R} \text{ and } x \geq 4\}$
 - ... $\{y: y \in \mathbb{R} \text{ and } y \geq 4\}$
 - ... \mathbb{R}
 - ... \mathbb{R}_0^+
- f cannot be a function if ...
 - ... the domain of f is no number set.
 - ... the codomain of f contains more elements than the domain of f .
 - ... the domain of f contains more elements than the codomain of f .
 - ... at least one element of the domain of f has more than one image.
- If the range of a function contains as many elements as the domain, it can be concluded that ...
 - ... the range is the same set as the domain.
 - ... the codomain contains as many elements as the domain.
 - ... each element of the codomain is also an element of the range.
 - ... no element of the range is associated to more than one element of the domain.

Answers

- 3.1 a) no function
No element (instead of exactly one element) of C is associated to one of the elements of D.
- b) function
- c) no function
Two elements (instead of exactly one element) of C are associated to one of the elements of D.
- d) function
- e) no function
More than one element (instead of exactly one element) of C are associated to some elements of D.
- f) no function
Many elements (instead of exactly one element) of C are associated to each element of D.
- g) function
- h) function
- i) no function
Two elements (instead of exactly one element) of \mathbb{R} are associated to each element of \mathbb{R}^+ .
- j) function

- 3.2 a) $R = \{A, D, F, J, M, N, O, S\}$
- b) $R = \{\text{Berlin, Vienna, Vaduz, Rome, Paris}\}$
- c) $R = C$
- d) $R = \mathbb{R}_0^+$

Notice:

- \mathbb{R}_0^+ is the set of all positive real numbers, including zero, i.e. $\mathbb{R}_0^+ = \{x: x \in \mathbb{R} \text{ and } x \geq 0\}$.

- 3.3 a) i) $f(1) = 1^3 - 1 = 0$
- ii) $f(-2) = (-2)^3 - (-2) = -6$
- iii) $f(a) = a^3 - a$
- iv) $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$
- v) $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$
- vi) $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$

- b) i) $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$
- ii) $g(-3) = \frac{(-3)^2}{-3+1} = -\frac{9}{2}$
- iii) $g(a) = \frac{a^2}{a+1}$
- iv) $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$
- v) $g(a - b) = \frac{(a-b)^2}{(a-b)+1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$
- vi) $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right)+1} = \frac{x^4}{x^3 + 2x^2 + 2x + 1}$

- 3.4
- a) $f(-1) = -2$
 - b) $f(2) \approx 2.8$
 - c) $x_1 = -3, x_2 = 1$
 - d) $x_1 \approx -2.5, x_2 \approx 0.3$
 - e) $D = \{x: x \in \mathbb{R} \text{ and } -3 \leq x \leq 3\} = [-3, 3]$
 - f) $R = \{y: y \in \mathbb{R} \text{ and } -2 \leq y \leq 3\} = [-2, 3]$
- 3.5
- a) 4th statement
 - b) 4th statement
 - c) 4th statement