

Exercises 2 Numbers Number sets, intervals, absolute value

Objectives

- know the definition and elements of natural numbers, integers, rational numbers, and real numbers.
- know and understand what an open, half-open, and closed interval is.
- know and understand what the absolute value of a real number is.
- be able to perform basic operations with real numbers.

Problems

2.1 Decide whether each statement is true or false:

- | | | |
|--------------------------------------|--|---|
| a) $4 \in \mathbb{N}$ | b) $-\frac{14}{7} \in \mathbb{Z}$ | c) $\sqrt{2} \in \mathbb{Q}$ |
| d) $\sqrt{9} \in \mathbb{N}$ | e) $\sqrt{9} \in \mathbb{Q}$ | f) $\sqrt{9} \in \mathbb{R}$ |
| g) $1.67854 \in \mathbb{Q}$ | h) $1.6\overline{7854} \in \mathbb{Q}$ | i) $\mathbb{N} \subset \mathbb{Z}$ |
| j) $\mathbb{Z} \subseteq \mathbb{Q}$ | k) $\mathbb{Q} \subset \mathbb{R}$ | l) $\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$ |

2.2 Determine the following sets:

- | | | |
|--|--|--|
| a) $\mathbb{Z} \setminus \mathbb{N}$ | b) $\mathbb{Z} \cup \mathbb{N}$ | c) $\mathbb{Z} \cap \mathbb{N}$ |
| d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ | e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$ | f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$ |

2.3 You will find a pdf-file with scanned pages of the textbook Harshbarger/Reynolds* on Moodle:
> Additional Materials > Algebraic Concepts (Harshbarger/Reynolds)
(pages 2 to 55 of chapter “0 Algebraic Concepts” and pages A1 to A5)

Go to section “0.2 The Real Numbers” (pages 9 to 15).

- Study the theory (pages 9 to 13).
- Do the odd-numbered exercises 1 to 45 (pages 13 and 14).

*Harshbarger, R.J., Reynolds, J.J.: Mathematical Applications for the Management, Life, and Social Sciences; Houghton Mifflin Company, Boston / New York 2007, 8th edition, ISBN 978-0-618-73162-6

2.4 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

- | | | |
|----|--------------------------|---|
| a) | <input type="checkbox"/> | $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$ |
| | <input type="checkbox"/> | $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$ |
| | <input type="checkbox"/> | $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$ |
| | <input type="checkbox"/> | $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, \dots\}$ |
| b) | <input type="checkbox"/> | $\mathbb{N} = [1, \infty)$ |
| | <input type="checkbox"/> | $3 \in (3, 4)$ |
| | <input type="checkbox"/> | $[3, 4] \cup (3, 4) = (3, 4)$ |
| | <input type="checkbox"/> | $[3, 4] \setminus (3, 4) = \{3, 4\}$ |
| c) | | (see next page) |

c) Assume that x is a rational number. Therefore, it can be concluded that x is ...

- ... a real number.
- ... an integer.
- ... a fraction where both numerator and denominator are natural numbers.
- ... a natural number.

Answers

- 2.1 a) true b) true c) false
 d) true e) true f) true
 g) true h) true i) true
 j) true k) true l) false

- 2.2 a) $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, \dots\}$
 b) $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$
 c) $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$
 d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$
 e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$
 f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$

- 2.3 see Harshbarger/Reynolds (page A1)

Note:

- Only answers of the odd-numbered exercises (1, 3, 5, ...) are available.

- 2.4 a) 3rd statement
 b) 4th statement
 c) 1st statement