Derivative
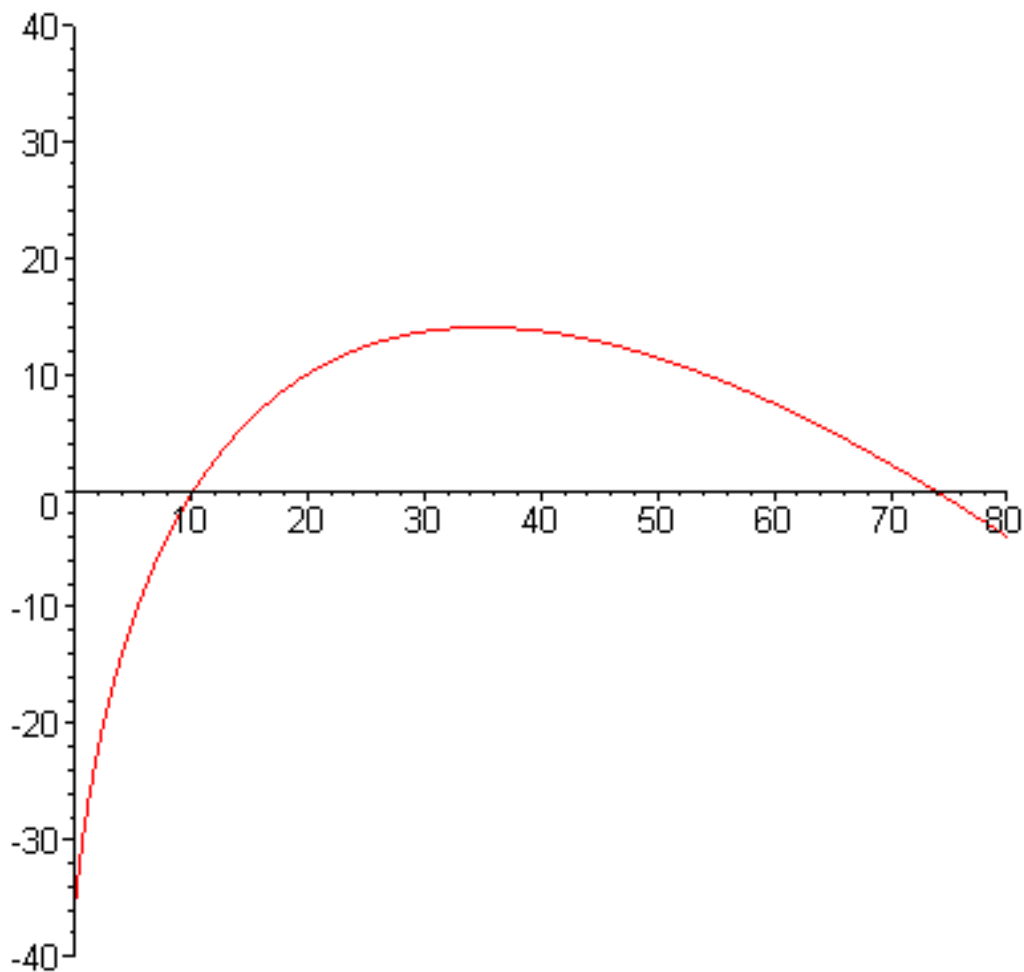

Function f

f: $D \to \mathbb{R}$ where $D \subseteq \mathbb{R}$
    $x \mapsto y = f(x)$

Ex.: $f(x) = 24\sqrt{x + 1} - 2x - 60$

What do we want to know?

  Slope of the tangent to the graph of the function f at a certain point $A(x_0 \mid f(x_0))$.

Why do we want to know the slope?

  - **increasing** (slope > 0), **decreasing** (slope < 0)
  - relative **maximum/minimum** (slope = 0)
  - **concavity** (concave up if slope increases, concave down if slope decreases), points of inflection


Applications in economics
- tendency of costs/revenue/profit
- maximum/minimum of costs/revenue/profit
- **marginal costs/revenue/profit** (change of costs/revenue/profit if number x of items increases by one)

Definition

The slope of the tangent to the graph of f through the point $A(x_0 \mid f(x_0))$ is called the **derivative** (or **rate of change**) **of f at x_0**, denoted $f'(x_0)$ ("f prime of x_0").

How can we determine the slope?
The slope of the **secant** through the points $A(x_0 \mid f(x_0))$ and $B(x_0+\Delta x \mid f(x_0+\Delta x))$ tends towards the slope of the **tangent** through $A(x_0 \mid f(x_0))$ as Δx tends towards 0.
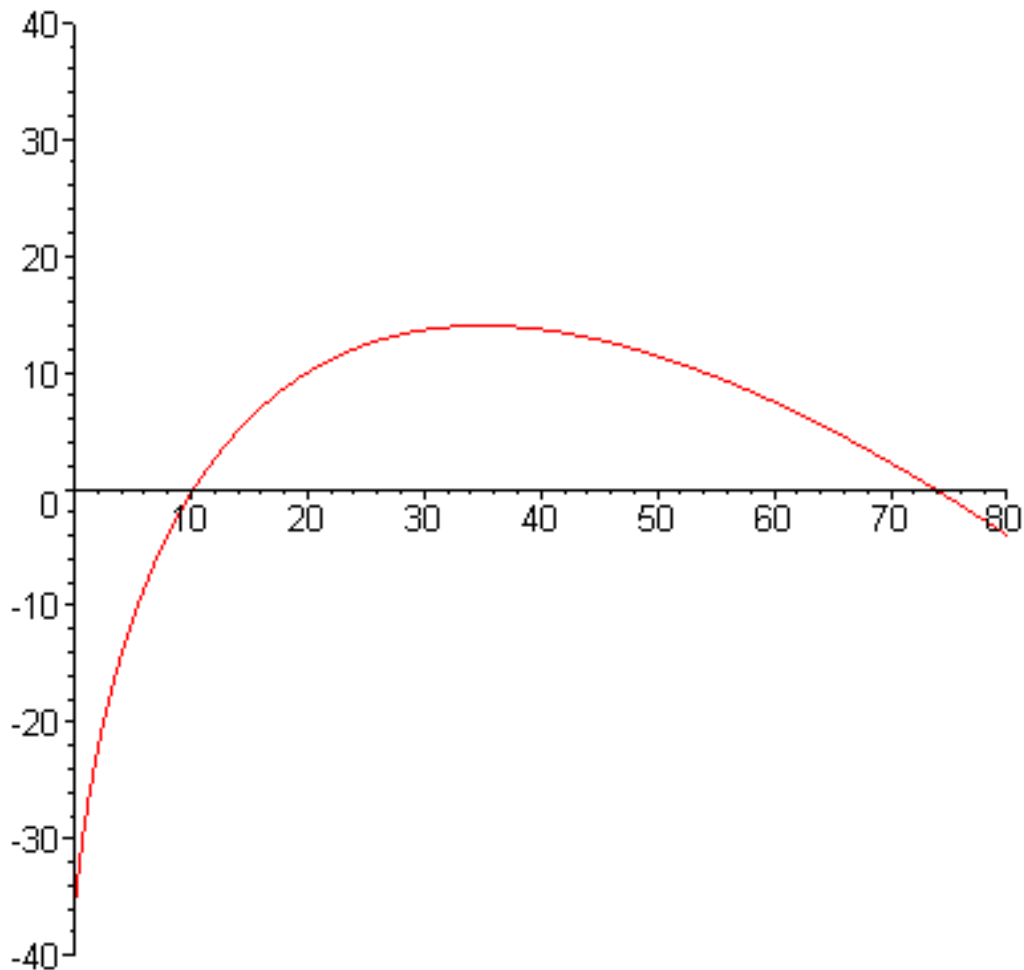
Ex.:   f: $\mathbb{R} \to \mathbb{R}$
             $x \mapsto y = f(x) = x^2$

         $f'(x_0) = 2x_0$

Definition

Suppose that the derivative (rate of change) $f'(x_0)$ exists for all $x_0 \in D_1$, where $D_1 \subseteq D$.

The function f'

$f' :\ D_1 \rightarrow \mathbb{R}$
$\qquad x \ \mapsto \ y = f'(x)$

is called the **derivative** (or **derived function**) **of f**.

Ex. 1: $\quad f :\ \mathbb{R} \rightarrow \mathbb{R}$ $\qquad\qquad\qquad\qquad$ $f' :\ \mathbb{R} \rightarrow \mathbb{R}$
$\qquad\qquad\quad x \mapsto \ y = f(x) = x^2$ $\qquad\qquad\quad x \mapsto \ y = f'(x) = 2x$

Ex. 2: $\quad f :\ D \rightarrow \mathbb{R}$ $\qquad\qquad\qquad\qquad\quad$ $f' :\ D_1 \rightarrow \mathbb{R}$
$\qquad\qquad\quad x \mapsto \ y = f(x) = 24\sqrt{x+1} - 2x - 60$ $\qquad\quad x \ \mapsto \ y = f'(x) = \dfrac{12}{\sqrt{x+1}} - 2$