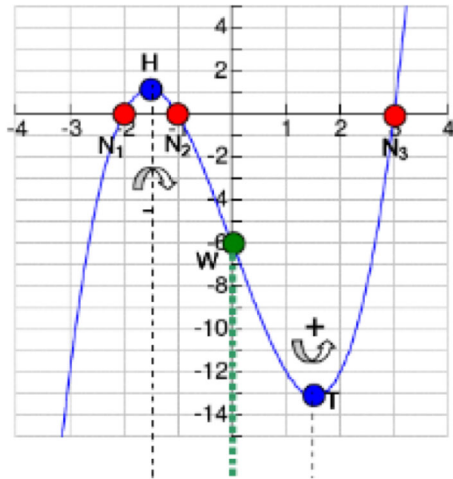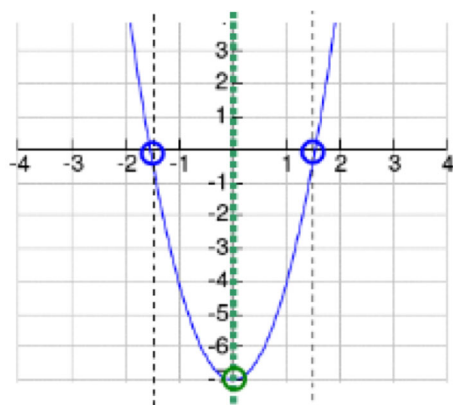# Increasing/decreasing, concavity

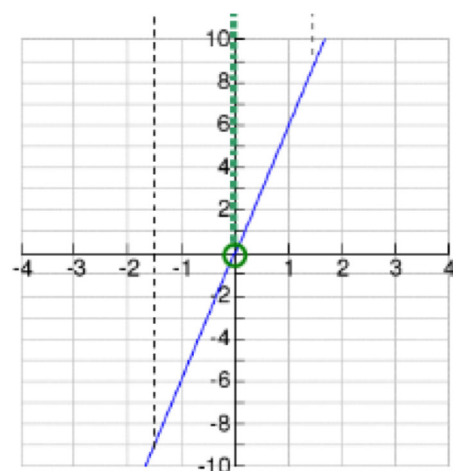Ex.: $f(x) = x^3 - 7x - 6$

$f'(x) = 3x^2 - 7$

$f''(x) = 6x$

Increasing/decreasing

The function f is **increasing** at $x = x_0$, if the **first derivative** is **positive**, i.e. $f'(x_0) > 0$.

The function f is **decreasing** at $x = x_0$, if the **first derivative** is **negative**, i.e. $f'(x_0) < 0$.

Concavity

The graph of the function f is **concave up** at $x = x_0$, if the **second derivative** is **positive**, i.e. $f''(x_0) > 0$.

The graph of the function f is **concave down** at $x = x_0$, if the **second derivative** is **negative**, i.e. $f''(x_0) < 0$.

Relative maxima/minima

The function f has a **relative maximum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave down at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) < 0$.

The function f has a **relative minimum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave up at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) > 0$.

Absolute maximum/minimum

The **absolute maximum/minimum** of a continuous function f is either a relative maximum/minimum or the value of f at one of the endpoints of the domain.

Points of inflection

The function f has a **point of inflection** at $x = x_0$, if the graph of f changes its concavity from concave up to concave down (or vice versa) at $x = x_0$, i.e. if $f''(x_0) = 0$ and $f'''(x_0) \neq 0$.

Ex.: $f(x) = x^3 - 7x - 6$ (see page 1) $\Rightarrow$ $f'(x) = 3x^2 - 7$

$\Rightarrow$ $f''(x) = 6x$

$\Rightarrow$ $f'''(x) = 6$

Relative maxima/minima

$f'(x) = 0$ at $x_1 = \sqrt{\frac{7}{3}} = 1.52\ldots$ and $x_2 = -\sqrt{\frac{7}{3}} = -1.52\ldots$

$f''(x_1) = 6 \cdot \sqrt{\frac{7}{3}} = 9.16\ldots > 0$ $\Rightarrow$ relative minimum at $x_1 = \sqrt{\frac{7}{3}}$

$f''(x_2) = -6 \cdot \sqrt{\frac{7}{3}} = -9.16\ldots < 0$ $\Rightarrow$ relative maximum at $x_2 = -\sqrt{\frac{7}{3}}$

Absolute maximum/minimum

Ex.: $D = [0,4]$ $\Rightarrow$ absolute maximum at $x = 4$ (endpoint of domain)

$\Rightarrow$ absolute minimum at $x = x_1 = \sqrt{\frac{7}{3}}$ (relative minimum)

Ex.: $D = [-4,3]$ $\Rightarrow$ absolute maximum at $x = x_2 = -\sqrt{\frac{7}{3}}$ (relative maximum)

$\Rightarrow$ absolute minimum at $x = -4$ (endpoint of domain)

Points of inflection

$f''(x) = 0$ at $x_3 = 0$

$f'''(x_3) = 6 \neq 0$ $\Rightarrow$ point of inflection at $x_3 = 0$

Financial mathematics

Marginal cost/revenue/profit function = first derivative of the cost/revenue/profit function

Ex.:   Cost function $C(x) = 120 + 2x^2$
        $\Rightarrow$ Marginal cost function $C'(x) = 4x$

        Revenue function $R(x) = 168x - x^2$
        $\Rightarrow$ Marginal revenue function $R'(x) = 168 - 2x$

        Profit function $P(x) = R(x) - C(x) = -120 + 168x - 3x^2$
        $\Rightarrow$ Marginal profit function $P'(x) = 168 - 6x$

Average cost/revenue/profit function

        Average cost function $\overline{C}(x) := \dfrac{C(x)}{x}$ where $C(x)$ = cost function

        Ex.:   Cost function $C(x) = 3x^2 + 4x + 2$
            $\Rightarrow$ Average cost function $\overline{C}(x) = 3x + 4 + \dfrac{2}{x}$

        Average revenue function $\overline{R}(x) := \dfrac{R(x)}{x}$ where $R(x)$ = revenue function

        Average profit function $\overline{P}(x) := \dfrac{P(x)}{x}$ where $P(x)$ = profit function