

R2.5 The total revenue function for a commodity is given by

$$R(x) = 36x - 0.01x^2$$

Find the maximum revenue ...

- a) ... if production is not limited to a certain number of units.
- b) ... if production is limited to at most 1500 units.

R2.6 If the total cost function for a product is

$$C(x) = 100 + x^2$$

producing how many units x will result in a minimum average cost per unit? Find the minimum average cost.

R2.7 A firm can produce only 1000 units per month. The monthly total cost is given by

$$C(x) = 300 + 200x$$

dollars, where x is the number produced. If the total revenue is given by

$$R(x) = 250x - \frac{1}{100}x^2$$

dollars, how many items should the firm produce for maximum profit? Find the maximum profit.

R2.8 Determine the indefinite integrals below:

a) $\int (x^4 - 3x^3 - 6) \, dx$

b) $\int \left(\frac{1}{2}x^6 - \frac{2}{3x^4} \right) \, dx$

R2.9 The equation of the third derivative f''' of a function f is given as follows:

$$f'''(x) = 3x + 1$$

Find the equation of the function f such that $f''(0) = 0$, $f'(0) = 1$, $f(0) = 2$

R2.10 If the marginal cost (in CHF) for producing a product is $C'(x) = 5x + 10$, with a fixed cost of 800 CHF, what will be the cost of producing 20 units?

R2.11 A certain firm's marginal cost $C'(x)$ and the derivative of the average revenue $\bar{R}'(x)$ are given as follows:

$$C'(x) = 6x + 60$$

$$\bar{R}'(x) = -1$$

The total cost and revenue of the production of 10 items are 1000 CHF and 1700 CHF, respectively.

How many units will result in a maximum profit? Find the maximum profit.

R2.12 The demand function for a product is

$$p = f(x) = 49 - x^2$$

and the supply function is

$$p = g(x) = 4x + 4$$

Find the equilibrium point and both the consumer's and the producer's surplus there.

R2.13 The demand function for a product is

$$p = f(x) = 110 - ax^2$$

and the supply function is

$$p = g(x) = 2 - \frac{6}{5}x + bx^2$$

with unknown parameters a and b . The equilibrium price is 10 CHF, and the producer's surplus is 73.33 CHF

Determine the two unknown parameters a and b .

b) $f(x) = 4x^2(x^2 - 1)$
 $f'(x) = 16x^3 - 8x = 8x(2x^2 - 1)$
 $f''(x) = 48x^2 - 8 = 8(6x^2 - 1)$
 $f'''(x) = 96x$

i) $f'(x) = 0$ at $x_1 = 0$, $x_2 = \frac{1}{\sqrt{2}}$, and $x_3 = -\frac{1}{\sqrt{2}}$
 $f''(x_1) = -8 < 0 \Rightarrow$ relative maximum at $x_1 = 0$
 $f''(x_2) = 16 > 0 \Rightarrow$ relative minimum at $x_2 = \frac{1}{\sqrt{2}}$
 $f''(x_3) = 16 > 0 \Rightarrow$ relative minimum at $x_3 = -\frac{1}{\sqrt{2}}$

ii) $f''(x) = 0$ at $x_3 = \frac{1}{\sqrt{6}}$
 $f'''(x_3) = \frac{96}{\sqrt{6}} \neq 0 \Rightarrow$ point of inflection at $x_3 = \frac{1}{\sqrt{6}}$

- R2.5 a) **Relative** maximum at $x_1 = 1800$
 $R(x_1) = 32'400$ CHF
 $R(x) < R(x_1)$ if $x \neq x_1$ as there is no relative minimum
 $\Rightarrow R = 32'400$ CHF is the **absolute** maximum revenue at $x = 1800$.
- b) Relative maximum at $x = 1800$ lies outside the possible interval $0 \leq x \leq 1500$
 $R(1500) = 31'500$ CHF $>$ $R(0) = 0$ CHF
 $\Rightarrow R = 31'500$ CHF is the **absolute** maximum revenue at $x = 1500$.

R2.6 $\bar{C}(x) = \frac{C(x)}{x} = \frac{100}{x} + x$
 $\bar{C}(x)$ has a **relative** minimum at $x_1 = 10$
 $\bar{C}(10) = 20$ CHF
 $\bar{C}(x) > \bar{C}(x_1)$ if $x \neq x_1$ as there is no relative maximum
 $\Rightarrow \bar{C} = 20$ CHF is the **absolute** minimum average cost at $x = 10$.

R2.7 $P(x) = R(x) - C(x) = -\frac{1}{100}x^2 + 50x - 300$
 $P(x)$ has a **relative** maximum at $x_1 = 2500$. This is outside the possible interval $0 \leq x \leq 1000$
 $P(1000) = 39'700$ CHF $>$ $P(0) = -300$ CHF
 $\Rightarrow P = 39'700$ CHF is the **absolute** maximum profit at the endpoint $x = 1000$.

R2.8 a) $\int (x^4 - 3x^3 - 6) dx = \frac{x^5}{5} + \frac{3x^4}{4} - 6x + C$

b) $\int \left(\frac{1}{2}x^6 - \frac{2}{3x^4} \right) dx = \frac{x^7}{14} + \frac{2}{9x^3} + C$

R2.9 $f(x) = \frac{x^4}{8} + \frac{x^3}{6} + x + 2$

R2.10 $C(20) = 2000$ CHF

Hint:

- First, determine the cost function $C(x) \Rightarrow C(x) = \frac{5}{2}x^2 + 10x + 800$

R2.11 $P = 800$ CHF is the absolute maximum profit at $x = 15$ units.

Hints:

- Determine the cost function $C(x) \Rightarrow C(x) = 3x^2 + 60x + 100$

- Determine the average revenue function $\bar{R}(x) \Rightarrow \bar{R}(x) = -x + C$
- Determine the revenue function $R(x) \Rightarrow R(x) = -x^2 + 180x$
- Find the profit function $P(x) \Rightarrow P(x) = -4x^2 + 120x - 100$
- Find the relative maximum of the profit function $P(x)$.
- Check if the relative maximum is the absolute maximum.

R2.12 Equilibrium quantity $x = 5$
 Equilibrium price $p = 24$
 Consumer's surplus $CS = 83.33$
 Producer's surplus $PS = 50$

R2.13 $a = 1$
 $b = 0.2$