Exercises 5   Linear function and equations
Linear equations

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to treat applied tasks in economics by means of linear equations.

Problems

5.1   Determine the solution sets of the following equations:

   a) $19x - 32 + 17x = 18x - 30 + 16x - 4$

   b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$

   c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$

   d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

5.2   Determine the solution sets of the following equations:

   a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$

   b) $184 - 6(x - 24) = 214 - 3(2x - 38)$

   c) $(x + 3)(x - 5) = (x - 3)^2$

   d) $(x - 5)(x - 2) = (x - 4)(x - 3)$

   e) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

5.3   Determine the solution sets of the following equations:

   a) $\dfrac{x + 3}{5} = \dfrac{2x - 8}{3}$

   b) $\dfrac{x + 3}{4} + \dfrac{1 - 3x}{7} = 0$

   c) $\dfrac{2}{x - 1} = \dfrac{1}{x - 2}$

   d) $\dfrac{x}{x - 1} = \dfrac{x - 1}{x - 2}$

5.4   The equations below are equations in the variable x. Furthermore, the equations contain real parameters a and b. Therefore, the solution sets of the equations depend on the values of those parameters.

   Solve the equations for x, and determine the solution sets.
   Take into account that the parameters a and b can be any real numbers.

   a) $x(a - 3) = a$

           Hint:
           - You may want to divide both sides of the equation by a - 3. However, this is not allowed if
             a - 3 = 0, i.e. if a = 3, as dividing by 0 is not defined.

   b) $(x + 1)(b - 2) = 2bx$

   c) $a(1 + x) - 1 = x(2a - 1)$

   d) $(a - b)x = a$

5.5   The graph of a linear function f with slope a contains the point P. Find the equation of the linear function.

        a)   a = -5   P(5|-3)

        b)   a = 2   P(3|0)

        c)   a = 0   P(2|3)

5.6   Alps Bikes uses the formula B(t) = - 400 t + 5000 to find the book value B(t), in Swiss franks, of a mountain bike t years after its purchase.

        a)   What do the numbers - 400 and 5000 signify?

        b)   How long will it take the mountain bike to depreciate completely?

5.7   Two items A and B depreciate linearly:

                Item A   original value = 200 CHF
                                    depreciation = 16 CHF/year

                Item B   original value = 240 CHF
                                    depreciation = 32 CHF/year

        a)   How long will it take the two items to depreciate completely?

        b)   Determine the point in time where both items have the same value.

        Hint:
        - Think of the temporal development of the values as linear functions.

5.8   Simple interest at an unknown rate is paid on an initial bank balance of 5000 CHF. The balance after five years is 5625 CHF.

        a)   Determine the interest rate.

        b)   How long would it take the balance to reach 7000 CHF?

5.9   Decide which statements are true or false. Put a mark into the corresponding box.
        In each problem a) to c), exactly one statement is true.

        a)   The solution set of a linear equation ...

                ☐   ... contains at least one element.
                ☐   ... never contains two elements.
                ☐   ... cannot contain elements if the linear equation does not correspond to a linear function.
                ☐   ... cannot be the empty set.

        b)   If a linear equation has exactly one solution ...

                ☐   ... the graph of the corresponding linear function intersects the x-axis.
                ☐   ... the equation does not contain any parameters.
                ☐   ... the solution set contains a rational number.
                ☐   ... no other linear function can have the same solution.

        c)   If a linear equation has the solution x = 2, it can be concluded that ...

                ☐   ... x = 3 is not a solution.
                ☐   ... the graph of the corresponding linear function intersects the x-axis at x = 2.
                ☐   ... P(2|0) is a point of the graph of the corresponding linear function.
                ☐   ... P(0|2) is a point of the graph of the corresponding linear function.

Answers

5.1   a)   S = {-1}

         b)   S = {13}

         c)   S = ℝ

         d)   S = { }

5.2   a) $S = \left\{\frac{11}{2}\right\}$

         b)   S = ℝ

         c)   S = {6}

         d)   S = { }

         e)   S = {0}

5.3   a)   S = {7}

         b)   S = {5}

         c)   S = {3}

         d)   S = { }

5.4   a)   if a = 3:   no solution $\Rightarrow$ S = { }

                if a ≠ 3: $x = \frac{a}{a-3}$ $\Rightarrow$ $S = \left\{\frac{a}{a-3}\right\}$

         b)   if b = -2:   no solution $\Rightarrow$ S = { }

                if b ≠ -2: $x = \frac{b-2}{b+2}$ $\Rightarrow$ $S = \left\{\frac{b-2}{b+2}\right\}$

         c)   if a = 1:   x ∈ ℝ $\Rightarrow$ S = ℝ

                if a ≠ 1:   x = 1 $\Rightarrow$ S = {1}

         d)   if a = b = 0:   x ∈ ℝ $\Rightarrow$ S = ℝ

                if a = b ≠ 0:   no solution $\Rightarrow$ S = { }

                if a ≠ b: $x = \frac{a}{a-b}$ $\Rightarrow$ $S = \left\{\frac{a}{a-b}\right\}$

5.5   a)   y = f(x) = - 5x + 22

                Hints:
                - The equation of a linear function is y = f(x) = ax + b
                - P(5|-3) is a point of the graph of the linear function. Therefore, the coordinates of P must fulfil the
                  equation of the linear function, i.e. -3 = f(5) = a·5 + b

         b)   y = f(x) = 2x - 6

         c)   y = f(x) = 3

5.6   a)   The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year.
                The number 5000 indicates that the original value of the mountain bike was 5000 CHF.

         b)   12.5 years

                Hint:
                - Complete depreciation at time t means B(t) = 0.

5.7   a)   item A: 12.5 years
                  item B: 7.5 years

        b) $t = 2.5$ years (if the original value is meant to be at $t = 0$ years)

5.8   a)   slope of the linear function: $a = 125$
                  interest rate $r = 2.5\%$

        b)   16 years

5.9   a)   2nd statement

        b)   1st statement

        c)   3rd statement