

## Exercises 2                      Numbers Number sets, intervals, absolute value

### Objectives

- know the definition and elements of the set of real numbers, rational numbers, integers, natural numbers.
- know and understand what an open, half-open, closed interval is.
- know and understand what the absolute value of a real number is.
- be able to perform basic operations with real numbers.

### Problems

2.1     Decide whether each statement is true or false:

- |                                      |  |   |
|--------------------------------------|--|---|
| a) $4 \in \mathbb{N}$                | b) $-\frac{14}{7} \in \mathbb{Z}$      | c) $\sqrt{2} \in \mathbb{Q}$                      |
| d) $\sqrt{9} \in \mathbb{N}$         | e) $\sqrt{9} \in \mathbb{Q}$           | f) $\sqrt{9} \in \mathbb{R}$                      |
| g) $1.67854 \in \mathbb{Q}$          | h) $1.6\overline{7854} \in \mathbb{Q}$ | i) $\mathbb{N} \subset \mathbb{Z}$                |
| j) $\mathbb{Z} \subseteq \mathbb{Q}$ | k) $\mathbb{Q} \subset \mathbb{R}$     | l) $\mathbb{R} \setminus \mathbb{Z} = \mathbb{N}$ |

2.2     Determine the following sets:

- |  |  |  |
|--|--|--|
| a) $\mathbb{Z} \setminus \mathbb{N}$                   | b) $\mathbb{Z} \cup \mathbb{N}$                        | c) $\mathbb{Z} \cap \mathbb{N}$                        |
| d) $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ | e) $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q})$ | f) $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N}$ |

2.3     Harshbarger/Reynolds\*: Chapter 0 (Algebraic Concepts), Section 0.2 (p. 9-15)  
(Scanned pages 2-55 and A1-A5 in file “Algebraic Concepts.pdf” on Moodle)

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|-------------------------|-----------------------------|
| a)     Theory (p. 9-13) | b)     Exercises (p. 13-15) |
|-------------------------|-----------------------------|

\*Harshbarger, R.J. and Reynolds, J.J.: Mathematical Applications for the Management, Life, and Social Sciences; Houghton Mifflin Company, Boston / New York 2007, 8th edition, ISBN 978-0-618-73162-6

2.4     Decide which statements are true or false. Put a mark into the corresponding box.  
In each problem a) to c), exactly one statement is true.

- a)       $\mathbb{N} \cup \mathbb{Z} = \mathbb{Q}$   
         $\mathbb{Q} \setminus \mathbb{Z} = \mathbb{N}$   
         $\mathbb{Q} \cap \mathbb{R} = \mathbb{Q}$   
         $\mathbb{Z} \setminus \mathbb{N} = \{-1, -2, -3, \dots\}$
- b)     Assume that  $x$  is a rational number. Therefore, it can be concluded that  $x$  is ...  
        ... a real number.  
        ... an integer.  
        ... a fraction where both numerator and denominator are natural numbers.  
        ... a natural number.
- c)       $\mathbb{N} = [1, \infty)$   
         $3 \in (3, 4)$   
         $[3, 4] \cup (3, 4) = (3, 4)$   
         $[3, 4] \setminus (3, 4) = \{3, 4\}$

**Answers**

- 2.1    a)    true                                    b)    true                                    c)    false  
          d)    true                                    e)    true                                    f)    true  
          g)    true                                    h)    true                                    i)    true  
          j)    true                                    k)    true                                    l)    false

- 2.2    a)     $\mathbb{Z} \setminus \mathbb{N} = \{0, -1, -2, -3, \dots\}$   
          b)     $\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$   
          c)     $\mathbb{Z} \cap \mathbb{N} = \mathbb{N}$   
          d)     $\mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q}) = \{\}$   
          e)     $\mathbb{Q} \cup (\mathbb{R} \setminus \mathbb{Q}) = \mathbb{R}$   
          f)     $(\mathbb{Q} \setminus \mathbb{Z}) \cap \mathbb{N} = \{\}$

- 2.3    see Harshbarger/Reynolds: Chapter 0, Algebraic Concepts  
          (Scanned pages 2-55 and A1-A5 in file “Algebraic Concepts.pdf” on Moodle)

- 2.4    a)    3<sup>rd</sup> statement  
          b)    1<sup>st</sup> statement  
          c)    4<sup>th</sup> statement