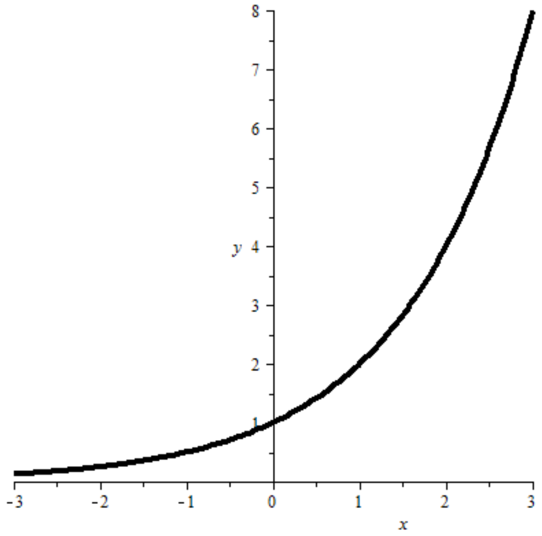
Exponential function

Definition

f:	$D \to \mathbb{R}$	$(D \subseteq \mathbb{R})$
	$x \mapsto y = f(x) = c \cdot a^x$	$(a \in \mathbb{R}^+ \setminus \{1\},\ c \in \mathbb{R} \setminus \{0\})$
	$a > 1$: exponential **growth**	
	$a < 1$: exponential **decay**	

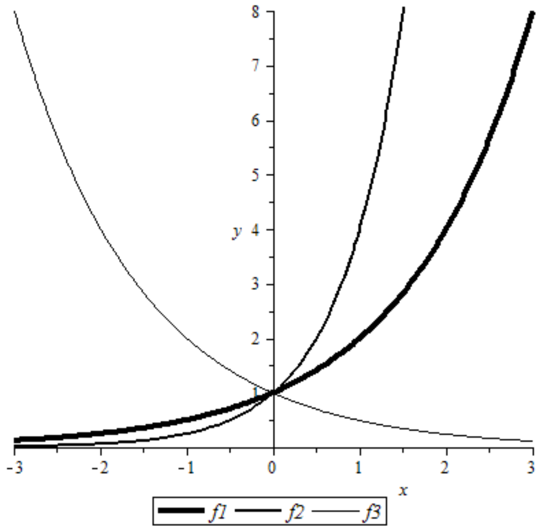
Graph

1. $y = f(x) = 2^x$ $(c = 1, a = 2)$



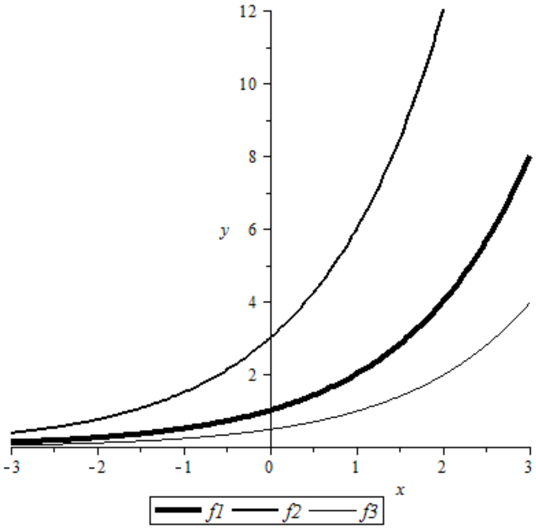
2. Parameter a (in all three cases below: c = 1)

$a = 2$: $y = f_1(x) = 2^x$
$a = 4$: $y = f_2(x) = 4^x$
$a = \frac{1}{2}$: $y = f_3(x) = \left(\frac{1}{2}\right)^x$

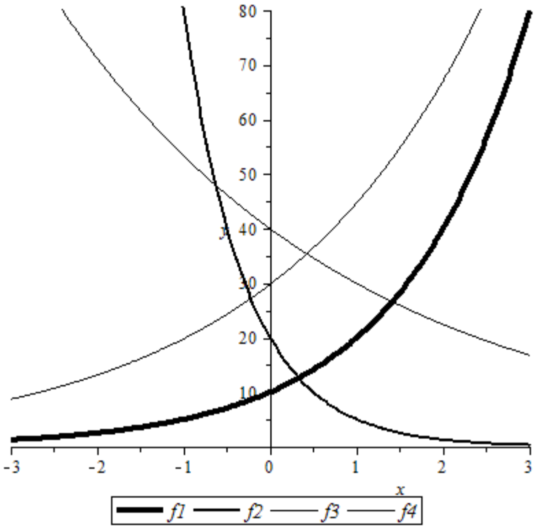


3. Parameter c (in all three cases below: $a = 2$)

$c = 1$: $y = f_1(x) = 2^x$
$c = 3$: $y = f_2(x) = 3 \cdot 2^x$
$c = \frac{1}{2}$: $y = f_3(x) = \frac{1}{2} \cdot 2^x$



4. $y = f_1(x) = 10 \cdot 2^x$ ($c = 10$, $a = 2$)
$y = f_2(x) = 20 \cdot 0.25^x$ ($c = 20$, $a = 0.25$)
$y = f_3(x) = 40 \cdot 0.75^x$ ($c = 40$, $a = 0.75$)
$y = f_4(x) = 30 \cdot 1.5^x$ ($c = 30$, $a = 1.5$)



Examples

1. Compound interest (exponential **growth**)

$C_n = C_0 \cdot q^n$

C_0 = initial capital
C_n = capital after n compounding periods
n = number of compounding periods (typically: 1 compounding period = 1 year)
q = growth factor = 1 + r (q > 1)
r = interest rate per compounding period

Ex.: $C_0 := 1000$, $r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$

2. Consumer price index (exponential **decay**)

$P(t) = P_0 \cdot q^t$

P_0 = initial purchasing power
P(t) = purchasing power at time t (typically: t in years)
q = decay factor (q < 1)

Ex.: $P_0 := 100$, $q := 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$