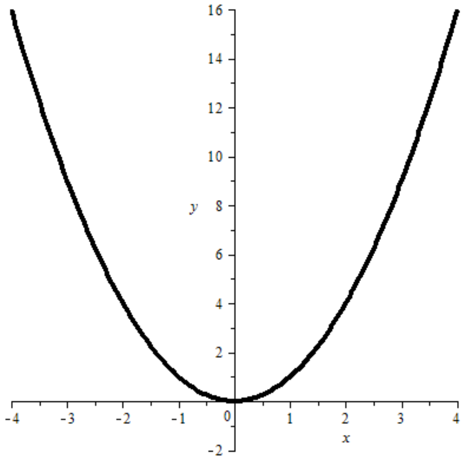
Quadratic function

Definition

f: D $\to$ $\mathbb{R}$	($D \subseteq \mathbb{R}$)
x $\mapsto$ $y = f(x) = ax^2 + bx + c$ general form	($a \in \mathbb{R}\setminus\{0\}$, $b \in \mathbb{R}$, $c \in \mathbb{R}$)
$y = f(x) = a(x - u)^2 + v$ vertex form	($a \in \mathbb{R}\setminus\{0\}$, $u \in \mathbb{R}$, $v \in \mathbb{R}$)

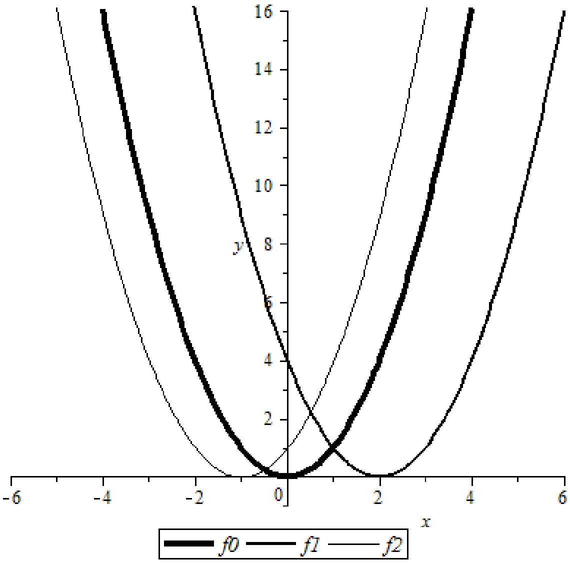
Graph

1. $y = f(x) = x^2$ $(a = 1, u = 0, v = 0)$



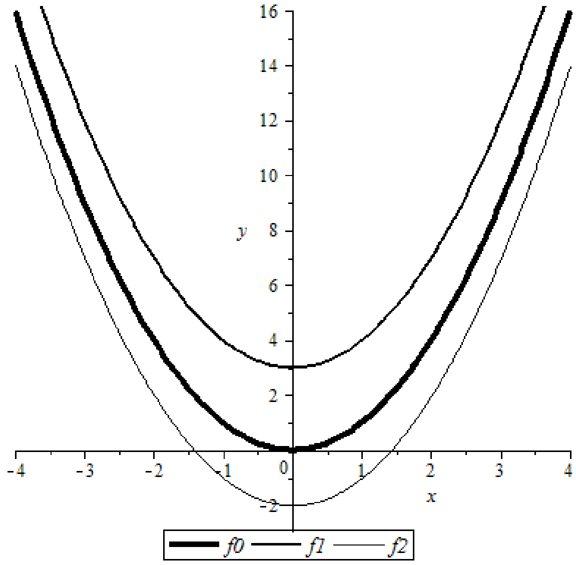
2. Parameter u (in all three cases below: $a = 1, v = 0$)

u = 0 : $y = f_0(x) = x^2$
u = 2 : $y = f_1(x) = (x - 2)^2$
u = -1 : $y = f_2(x) = (x + 1)^2$



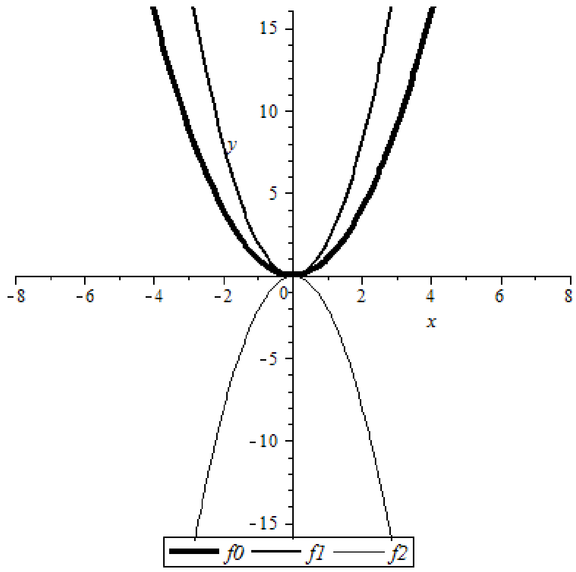
3. Parameter v (in all three cases below: a = 1, u = 0)

v = 0 : $y = f_0(x) = x^2$
v = 3 : $y = f_1(x) = x^2 + 3$
v = -2 : $y = f_2(x) = x^2 - 2$



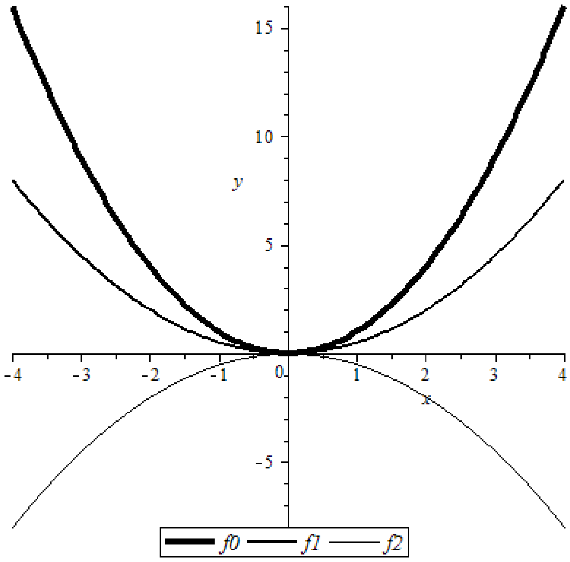
4. Parameter a (in all three cases below: u = 0, v = 0)

a = 1 : $y = f_0(x) = x^2$
a = 2 : $y = f_1(x) = 2x^2$
a = -2 : $y = f_2(x) = -2x^2$



5. Parameter a (in all three cases below: u = 0, v = 0)

$a = 1$: $y = f_0(x) = x^2$
$a = \frac{1}{2}$: $y = f_1(x) = \frac{1}{2}x^2$
$a = -\frac{1}{2}$: $y = f_2(x) = -\frac{1}{2}x^2$

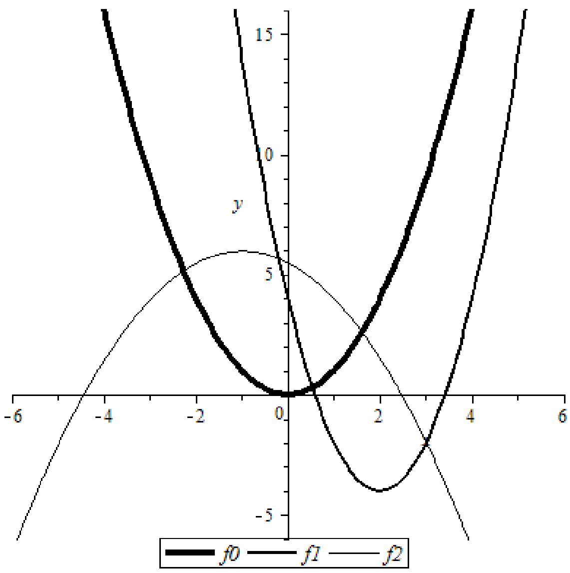


6. The **graph** of a quadratic function is a **parabola**.

The paramter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** V of the parabola: V(u|v)

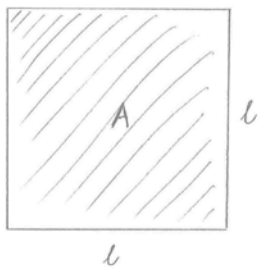
$y = f_0(x) = x^2$ (a = 1, u = 0, v = 0) V(0|0)
$y = f_1(x) = 2(x-2)^2 - 4$ (a = 2, u = 2, v = -4) V(2|-4)
$y = f_2(x) = -\frac{1}{2}(x+1)^2 + 6$ $\left(a = -\frac{1}{2}, u = -1, v = 6\right)$ V(-1|6)



Examples

1. Nature/Physics: Trajectories of water in a fountain

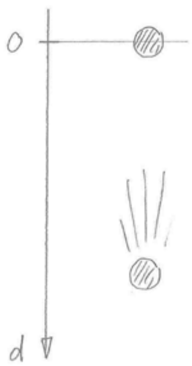
2. Geometry: Square



Area A for side length l: $A = l^2$

$f: \mathbb{R}^+ \rightarrow \mathbb{R}$
$l \mapsto A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t: $d = \frac{1}{2}gt^2$ (g = gravity field strength)

$f: \mathbb{R} \rightarrow \mathbb{R}$
$t \mapsto d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand