

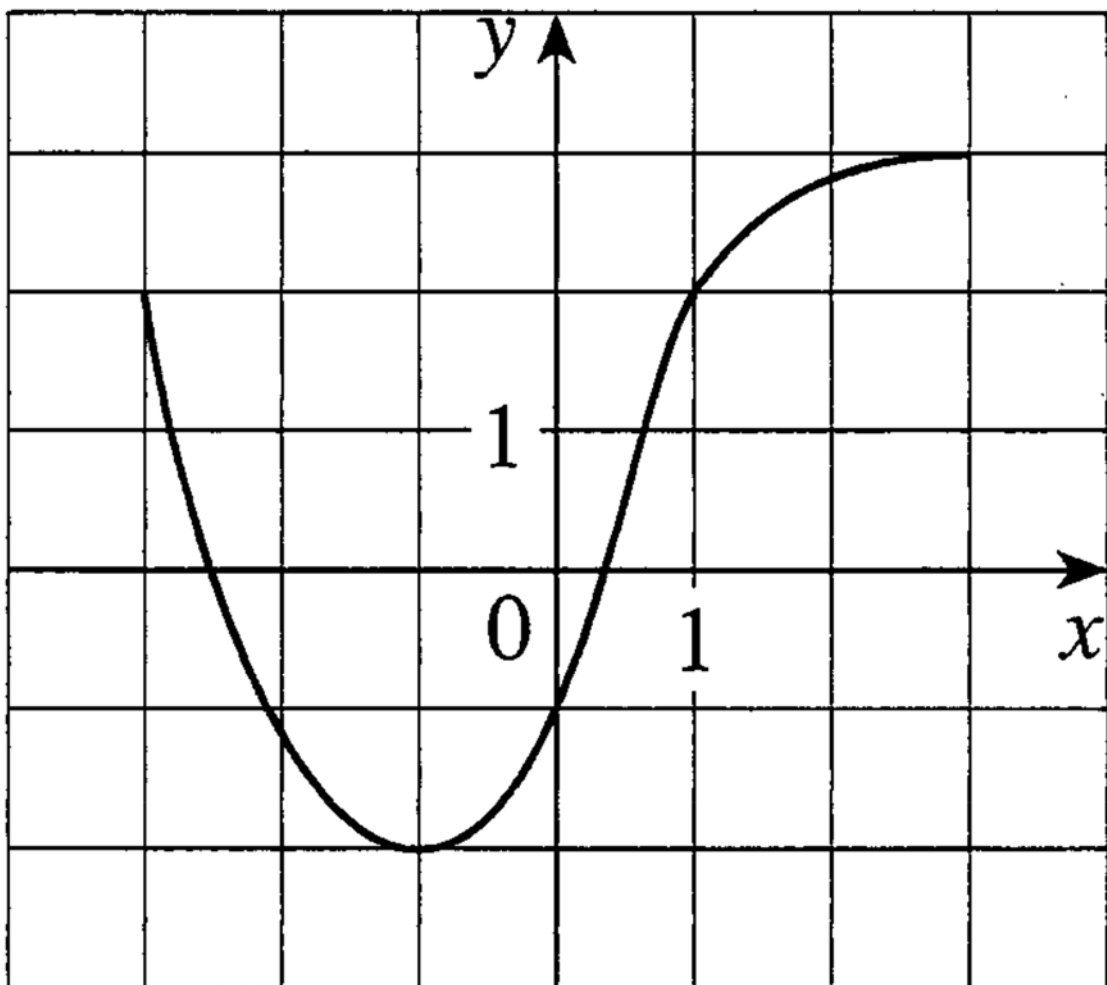
Exercises 13 Derivative Rate of change, derivative of constant/power/exponential functions

Objectives

- be able to estimate a rate of change out of the graph of a function.
- be able to state the rate of change of a constant/linear function.
- be able to determine a rate of change of a basic power/exponential function.
- be able to determine the derivative of a constant/linear function.
- be able to determine the derivative of a basic power/exponential function.

Problems

13.1 The graph of a function f is given as follows:



Estimate the rate of change $f'(x_0)$ at the given position x_0 :

- | | |
|---------------|---------------|
| a) $x_0 = -1$ | b) $x_0 = 0$ |
| c) $x_0 = 1$ | d) $x_0 = -2$ |

Hints:

- Draw the tangent to the graph of f at the given x_0 .
- Estimate the slope of the tangent.

13.2 The graph of a constant or linear function is a straight line. Therefore, the “tangent” to the graph at any point of the graph is the graph itself.

For each of the following functions $f: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto y = f(x) = \dots$

- i) ... draw the graph of f .
- ii) ... state the rate of change $f'(x_0)$ at the given x_0 .
- a) $f(x) = 3$ $x_0 = 2$
- b) $f(x) = c$ ($c \in \mathbb{R}$) any $x_0 \in \mathbb{R}$
- c) $f(x) = 2x - 3$ $x_0 = 4$
- d) $f(x) = mx + q$ ($m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R}$) any $x_0 \in \mathbb{R}$
- e) * $f(x) = |x|$ any $x_0 \in \mathbb{R}$

13.3 Determine $f'(x)$:

- | | | |
|-------------------------|---------------------------|------------------------------|
| a) $f(x) = 3$ | b) $f(x) = 0$ | c) $f(x) = -1$ |
| d) $f(x) = x^3$ | e) $f(x) = x^4$ | f) $f(x) = x^5$ |
| g) $f(x) = x^{17}$ | h) $f(x) = x^{200}$ | i) $f(x) = x^{100001}$ |
| j) $f(x) = x^{-1}$ | k) $f(x) = x^{-2}$ | l) $f(x) = x^{-17}$ |
| m) $f(x) = \frac{1}{x}$ | n) $f(x) = \frac{1}{x^3}$ | o) $f(x) = \frac{1}{x^{99}}$ |

13.4 Determine $f'(x)$:

- | | | |
|--|--|--|
| a) $f(x) = 3^x$ | b) $f(x) = 5^x$ | c) $f(x) = 18^x$ |
| d) $f(x) = \left(\frac{2}{3}\right)^x$ | e) $f(x) = \left(\frac{13}{17}\right)^x$ | f) $f(x) = \left(\frac{1}{4}\right)^x$ |
| g) $f(x) = \left(\frac{1}{e}\right)^x$ | h) $f(x) = \left(\frac{3}{e}\right)^x$ | i) $f(x) = \left(\frac{e}{3}\right)^x$ |

13.5 Determine the rate of change of the function f at the indicated x_0 :

- | | | |
|--|--------------------------|---------------------------|
| a) $f(x) = x$ | ii) $x_0 = 1$ | iii) $x_0 = -2$ |
| i) $x_0 = 0$ | | |
| b) $f(x) = x^5$ | ii) $x_0 = 2$ | iii) $x_0 = -\frac{2}{3}$ |
| i) $x_0 = 0$ | | |
| c) $f(x) = x^{-4}$ | ii) $x_0 = -\frac{4}{3}$ | iii) $x_0 = 0$ |
| i) $x_0 = -1$ | | |
| d) $f(x) = \left(\frac{2}{3}\right)^x$ | ii) $x_0 = 1$ | iii) $x_0 = -2$ |
| i) $x_0 = 0$ | | |

13.6 * (see next page)

- 13.6 * The rate of change $f'(x_0)$ of f at x_0 can be determined by looking at the secant through the points $A(x_0 | f(x_0))$ and $B(x_0 + \Delta x | f(x_0 + \Delta x))$ of the graph of f . The slope of this secant tends towards the slope of the tangent at $A(x_0 | f(x_0))$ as Δx tends towards 0.

It has been shown in class how to determine $f'(x_0)$ for the quadratic function $f(x) = x^2$.

Find $f'(x_0)$ for the following functions f :

a) $f(x) = x^3$ b) $f(x) = \frac{1}{x^2}$

- 13.7 Decide which statements are true or false. Put a mark into the corresponding box. In each problem a) to c), exactly one statement is true.

a) The rate of change of the function f at the position x_0 is a ...

- ... real number.
- ... function.
- ... tangent.
- ... graph.

b) The derivative f' of the function f is a ...

- ... real number.
- ... function.
- ... tangent.
- ... graph.

c) $f'(x_0)$ is the slope of the ...

- ... secant through the points $(0|0)$ and $(x_0|f(x_0))$.
- ... secant through the points $(x_0 + \Delta x | f(x_0 + \Delta x))$ and $(x_0 | f(x_0))$.
- ... tangent to the graph of f at $(x_0 | f(x_0))$.
- ... tangent to the graph of f' at $(x_0 | f(x_0))$.

Answers

- 13.1 a) $f'(-1) \approx 0$ b) $f'(0) \approx 2$
c) $f'(1) \approx \frac{3}{2}$ d) $f'(-2) \approx -\frac{5}{3}$
- 13.2 a) i) ...
ii) $f'(2) = 0$
b) i) ...
ii) $f'(x_0) = 0$
c) i) ...
ii) $f'(4) = 2$
d) i) ...
ii) $f'(x_0) = m$
e) * i) ...
ii) $f'(x_0) = \begin{cases} 1 & (x_0 > 0) \\ -1 & (x_0 < 0) \\ \text{not defined} & (x_0 = 0) \end{cases}$
- 13.3 a) $f'(x) = 0$ b) $f'(x) = 0$ c) $f'(x) = 0$
d) $f'(x) = 3x^2$ e) $f'(x) = 4x^3$ f) $f'(x) = 5x^4$
g) $f'(x) = 17x^{16}$ h) $f'(x) = 200x^{199}$ i) $f'(x) = 100'001x^{100'000}$
j) $f'(x) = -x^{-2}$ k) $f'(x) = -2x^{-3}$ l) $f'(x) = -17x^{-18}$
m) $f'(x) = -\frac{1}{x^2}$ n) $f'(x) = -\frac{3}{x^4}$ o) $f'(x) = -\frac{99}{x^{100}}$
- 13.4 a) $f'(x) = 3^x \ln(3)$ b) $f'(x) = 5^x \ln(5)$ c) $f'(x) = 18^x \ln(18)$
d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$ e) $f'(x) = \left(\frac{13}{17}\right)^x \ln\left(\frac{13}{17}\right)$
f) $f'(x) = \left(\frac{1}{4}\right)^x \ln\left(\frac{1}{4}\right) = -\frac{\ln(4)}{4^x}$
- Hint:
- Logarithm rules (see formulary) can be applied in order to simplify the result.
- g) $f'(x) = -\frac{1}{e^x}$ h) $f'(x) = \left(\frac{3}{e}\right)^x (\ln(3) - 1)$ i) $f'(x) = \left(\frac{e}{3}\right)^x (1 - \ln(3))$
- 13.5 a) $f'(x) = 1$
i) $f'(0) = 1$ ii) $f'(1) = 1$ iii) $f'(-2) = 1$
b) $f'(x) = 5x^4$
i) $f'(0) = 0$ ii) $f'(2) = 80$ iii) $f'\left(-\frac{2}{3}\right) = \frac{80}{81}$
c) $f'(x) = -\frac{4}{x^5}$
i) $f'(-1) = 4$ ii) $f'\left(-\frac{4}{3}\right) = \frac{243}{256}$ iii) $f'(0)$ is not defined
d) (see next page)

d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$

i) $f'(0) = \ln\left(\frac{2}{3}\right)$ ii) $f'(1) = \frac{2}{3} \ln\left(\frac{2}{3}\right)$ iii) $f'(-2) = \frac{9}{4} \ln\left(\frac{2}{3}\right)$

13.6 * a) ... b) ...

13.7 a) 1st statement
b) 2nd statement
c) 3rd statement