

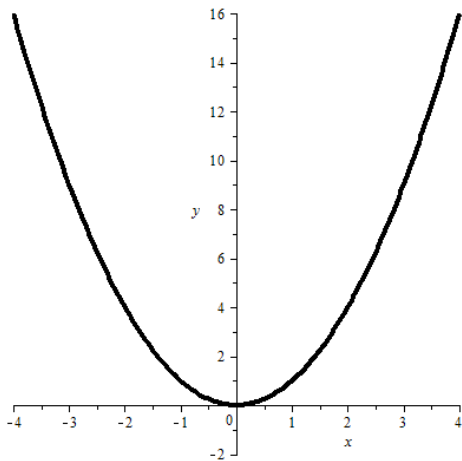
Quadratic function

Definition

$f: D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \mapsto y = f(x) = ax^2 + bx + c$ general form	$(a \in \mathbb{R} \setminus \{0\}, b \in \mathbb{R}, c \in \mathbb{R})$
$y = f(x) = a(x - u)^2 + v$ vertex form	$(a \in \mathbb{R} \setminus \{0\}, u \in \mathbb{R}, v \in \mathbb{R})$

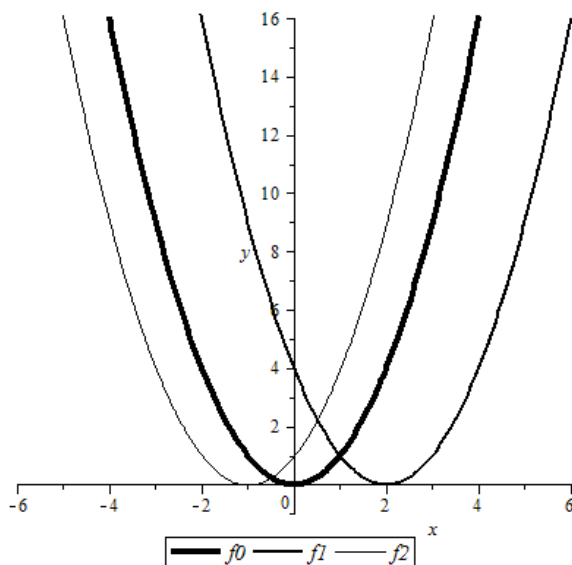
Graph

1. $y = f(x) = x^2$ ($a = 1, u = 0, v = 0$)



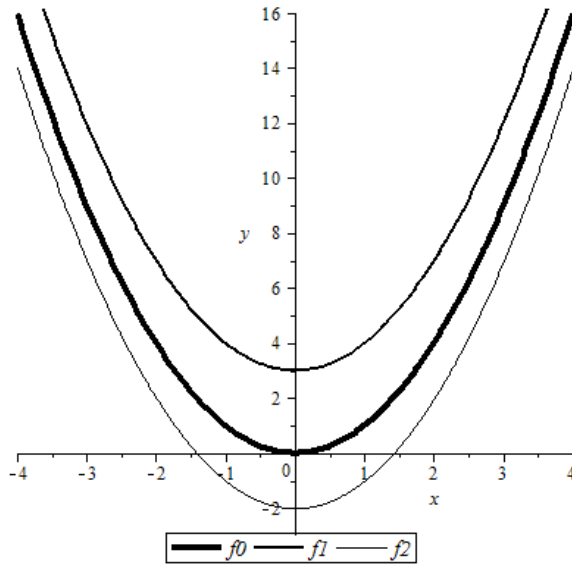
2. Parameter u (in all three cases below: $a = 1, v = 0$)

$u = 0$: $y = f_0(x) = x^2$
 $u = 2$: $y = f_1(x) = (x - 2)^2$
 $u = -1$: $y = f_2(x) = (x + 1)^2$



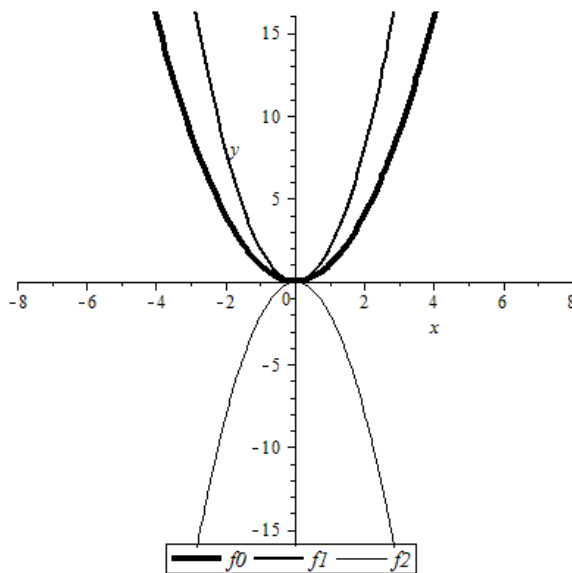
3. Parameter v (in all three cases below: $a = 1, u = 0$)

$$\begin{aligned} v = 0 : & \quad y = f_0(x) = x^2 \\ v = 3 : & \quad y = f_1(x) = x^2 + 3 \\ v = -2 : & \quad y = f_2(x) = x^2 - 2 \end{aligned}$$



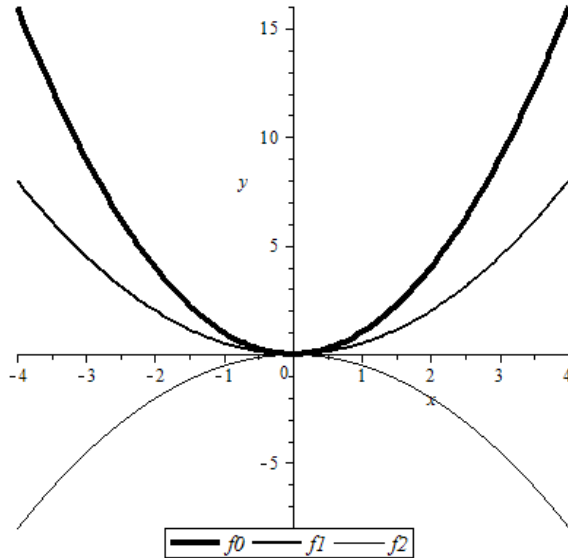
4. Parameter a (in all three cases below: $u = 0, v = 0$)

$$\begin{aligned} a = 1 : & \quad y = f_0(x) = x^2 \\ a = 2 : & \quad y = f_1(x) = 2x^2 \\ a = -2 : & \quad y = f_2(x) = -2x^2 \end{aligned}$$



5. Parameter a (in all three cases below: $u = 0, v = 0$)

$$\begin{aligned}
 a = 1 : & \quad y = f_0(x) = x^2 \\
 a = \frac{1}{2} : & \quad y = f_1(x) = \frac{1}{2}x^2 \\
 a = -\frac{1}{2} : & \quad y = f_2(x) = -\frac{1}{2}x^2
 \end{aligned}$$

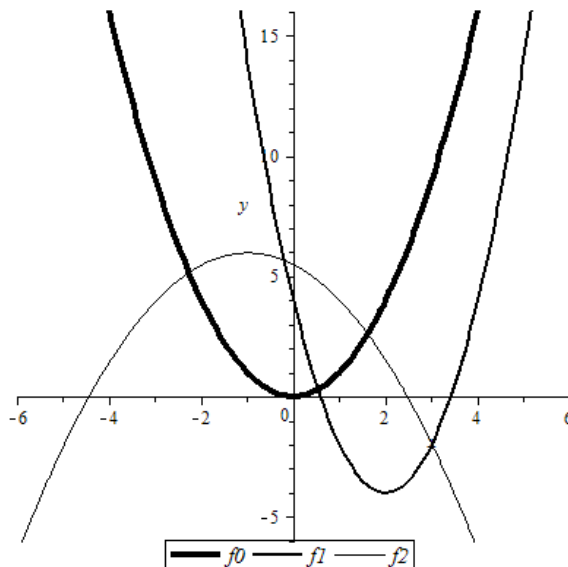


6. The **graph** of a quadratic function is a **parabola**.

The parameter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex V** of the parabola: $V(u|v)$

$y = f_0(x) = x^2$	$(a = 1, u = 0, v = 0)$	$V(0 0)$
$y = f_1(x) = 2(x - 2)^2 - 4$	$(a = 2, u = 2, v = -4)$	$V(2 -4)$
$y = f_2(x) = -\frac{1}{2}(x + 1)^2 + 6$	$(a = -\frac{1}{2}, u = -1, v = 6)$	$V(-1 6)$

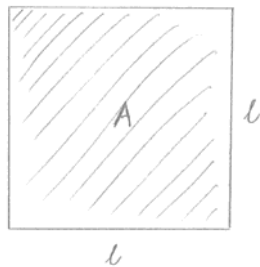


Examples

1. Nature/Physics: Trajectories of water in a fountain



2. Geometry: Square

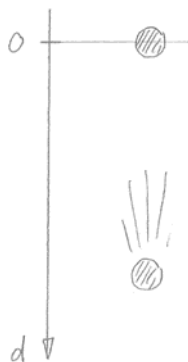


Area A for side length l : $A = l^2$

$f: \mathbb{R}^+ \rightarrow \mathbb{R}$

$l \mapsto A = f(l) = l^2$ quadratic function

3. Physics: Free fall



Distance d after time t : $d = \frac{1}{2}gt^2$ (g = gravity field strength)

$f: \mathbb{R} \rightarrow \mathbb{R}$

$t \mapsto d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand