

Exercises 17 **Definite integral** **Definite integral, area under a curve, consumer's/producer's surplus**

Objectives

- be able to determine the definite integral of a constant/basic power/basic exponential function.
- be able to determine the area between the graph of a basic power function and the abscissa.
- be able to determine the consumer's/producer's surplus if the demand and supply functions are basic power functions.

Problems

17.1 Calculate the definite integrals below:

a) $\int_3^4 (2x - 5) dx$ b) $\int_0^1 (x^3 + 2x) dx$ c) $\int_{-5}^{-3} \left(\frac{x^2}{2} - 4\right) dx$
d) $\int_2^4 \left(x^3 - \frac{x^2}{2} + 3x - 4\right) dx$ e) $\int_{-2}^2 \left(2x^2 - \frac{x^4}{8}\right) dx$ f) $\int_{-1}^1 e^x dx$

17.2 Determine the area between the graph of the function and the x-axis on the interval where the graph of f is above the x-axis, i.e. where $f(x) \geq 0$.

a) $f(x) = -x^2 + 1$ b) $f(x) = x^3 - x^2 - 2x$

17.3 The demand function for a product is $p = f(x) = 100 - 4x^2$.
If the equilibrium quantity is 4 units, what is the consumer's surplus?

17.4 The demand function for a product is $p = f(x) = 34 - x^2$.
If the equilibrium price is \$9, what is the consumer's surplus?

17.5 The demand function for a certain product is

$$p = f(x) = 81 - x^2$$

and the supply function is

$$p = g(x) = x^2 + 4x + 11.$$

Find the equilibrium point and the consumer's surplus there.

17.6 Suppose that the supply function for a good is $p = g(x) = 4x^2 + 2x + 2$.
If the equilibrium price is \$422, what is the producer's surplus?

17.7 Find the producer's surplus for a product if its demand function is

$$p = f(x) = 81 - x^2$$

and its supply function is

$$p = g(x) = x^2 + 4x + 11$$

17.8 The demand function for a certain product is

$$p = f(x) = 144 - 2x^2$$

and the supply function is

$$p = g(x) = x^2 + 33x + 48$$

Find the producer's surplus at the equilibrium point.

17.9 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The definite integral of a function is a ...

- ... real number.
- ... function.
- ... set of functions.
- ... graph.

b) $\int_a^b f(x) dx$...

- ... = $F(a) - F(b)$ where F is an antiderivative of f .
- ... is equal to the area between the graph of f and the x -axis in the interval $[a,b]$ if $f(x) \geq 0$ for all $x \in [a,b]$
- ... = 0 only if $f(x) = 0$ for all $x \in [a,b]$
- ... cannot be calculated unless all antiderivatives of f are known.

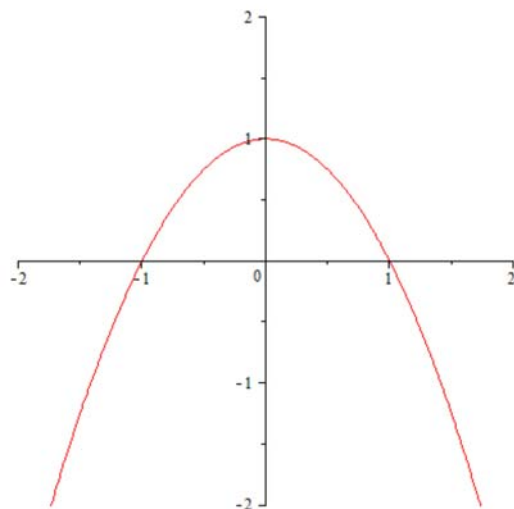
c) The consumer's surplus is an area between ...

- ... the graphs of the demand and the supply functions.
- ... the x axis and the graph of the demand function.
- ... the graph of the demand function and the horizontal line "price = equilibrium price".
- ... the horizontal line "price = equilibrium price" and the graph of the supply function.

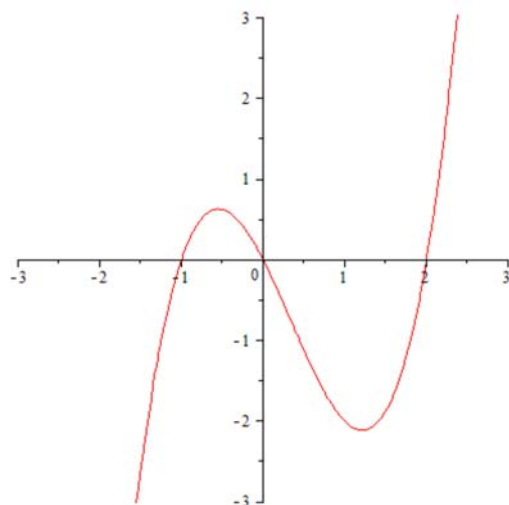
Answers

- 17.1 a) $\int_3^4 (2x - 5) dx = [x^2 - 5x]_3^4 = (4^2 - 5 \cdot 4) - (3^2 - 5 \cdot 3) = 2$
- b) $\int_0^1 (x^3 + 2x) dx = \left[\frac{x^4}{4} + x^2 \right]_0^1 = \left(\frac{1^4}{4} + 1^2 \right) - \left(\frac{0^4}{4} + 0^2 \right) = \frac{5}{4}$
- c) $\int_{-5}^{-3} \left(\frac{x^2}{2} - 4 \right) dx = \left[\frac{x^3}{6} - 4x \right]_{-5}^{-3} = \left(\frac{(-3)^3}{6} - 4 \cdot (-3) \right) - \left(\frac{(-5)^3}{6} - 4 \cdot (-5) \right) = \frac{25}{3}$
- d) $\int_2^4 \left(x^3 - \frac{x^2}{2} + 3x - 4 \right) dx = \left[\frac{x^4}{4} - \frac{x^3}{6} + \frac{3x^2}{2} - 4x \right]_2^4 = \left(\frac{4^4}{4} - \frac{4^3}{6} + \frac{3 \cdot 4^2}{2} - 4 \cdot 4 \right) - \left(\frac{2^4}{4} - \frac{2^3}{6} + \frac{3 \cdot 2^2}{2} - 4 \cdot 2 \right) = \frac{182}{3}$
- e) $\int_{-2}^2 \left(2x^2 - \frac{x^4}{8} \right) dx = \left[\frac{2x^3}{3} - \frac{x^5}{40} \right]_{-2}^2 = \left(\frac{2 \cdot 2^3}{3} - \frac{2^5}{40} \right) - \left(\frac{2 \cdot (-2)^3}{3} - \frac{(-2)^5}{40} \right) = \frac{136}{15}$
- f) $\int_{-1}^1 e^x dx = [e^x]_{-1}^1 = e^1 - e^{-1} = e - \frac{1}{e}$

17.2 a) $A = \int_{-1}^1 (-x^2 + 1) dx = \left[-\frac{x^3}{3} + x \right]_{-1}^1 = \frac{4}{3}$



b) $A = \int_{-1}^0 (x^3 - x^2 - 2x) dx = \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2 \right]_{-1}^0 = \frac{5}{12}$



Hints:

- First, find the positions x where the graph of f intersects the x -axis, i.e. where $f(x) = 0$
- Then, find the interval on which the graph of f is above the x -axis, i.e. where $f(x) \geq 0$

- 17.3 Consumer's surplus CS = \$170.67
- 17.4 Consumer's surplus CS = \$83.33
- 17.5 Equilibrium quantity $x = 5$
Equilibrium price $p = \$56$
Consumer's surplus CS = \$83.33
- 17.6 Producer's surplus PS = \$2766.67
- 17.7 Producer's surplus PS = \$133.33
- 17.8 Producer's surplus PS = \$103.34
- 17.9 a) 1st statement
b) 2nd statement
c) 3rd statement