Exercises 11 Exponential function and equations Compound interest, nominal/effective annual interest rate

Objectives

- be able to calculate the future capital that is invested at an interest rate which is compounded more than once per year.
- know and understand the terms "nominal annual interest rate" and "effective annual interest rate".
- be able to treat specific compound interest tasks.

Problems

11.1 An initial capital C_0 = 1000 CHF is invested at a nominal annual interest rate r = 10%, compounded ...

a) ... quarterly.

i) Determine the capitals C_1, C_2, and C_3, after one, two, and three years respectively.

ii) Determine the effective annual interest rate r*.

b) ... monthly.

i) Determine the capitals C_1, C_2, and C_3, after one, two, and three years respectively.

ii) Determine the effective annual interest rate r*.

11.2 Determine the effective annual interest rate for a nominal annual interest rate of 6%, compounded ...

a) ... annually.

b) ... semiannually.

c) ... quarterly.

d) ... monthly.

e) ... daily (1 year = 360 days).

11.3 What is the future value if $3200 is invested for 5 years at 8% compounded quarterly?

11.4 Find the interest that will be earned if $10'000 is invested for 3 years at 9% compounded monthly.

11.5 What amount of money do parents need to deposit in an account earning 10%, compounded monthly, so that it will grow to $40'000 for their son's college tuition in 18 years?

11.6 An initial capital of 1000 CHF amounts to 1500 CHF if it is invested for 10 years at an unknown annual interest rate, compounded quarterly.

Determine the ...

a) ... nominal annual interest rate.

b) ... effective annual interest rate.

11.7 How long (in months) would a capital have to be invested at 6%, compounded monthly, to double its value?

11.8 Ms P. wants to invest 100'000 CHF. Her bank makes two offers:

A effective annual interest rate of 8.5%

B nominal annual interest rate of 8%, compounded monthly

Which offer is better, offer A or offer B?

11.9 How long (in years) would 1000 CHF have to be invested at 2.5%, compounded daily, to earn 250 CHF interest?

11.10 At what nominal rate, compounded quarterly, would $20'000 have to be invested to amount to $26'425.82 in 7 years?

11.11 A couple needs $15'000 as a down payment for a home. If they invest the $10'000 they have at 8% compounded quarterly, how long will it take for the money to grow into $15'000?

11.12 Decide which statements are true or false. Put a mark into the corresponding box.
In each problem a) to c), exactly one statement is true.

a) The nominal interest rate ...

- ☐ ... is generally higher than the effective interest rate.
- ☐ ... is equal to the effective interest rate if interest is compounded annually.
- ☐ ... is half as much as the effective interest rate if interest is compounded semiannually.
- ☐ ... depends on the compounding period.

b) In a compound interest scheme where interest is compounded m (m > 1) times per year ...

- ☐ ... the growth factor is m times as high as if interest is compounded only once per year.
- ☐ ... the annual interest rate is m times lower than if interest is compounded only once per year.
- ☐ ... the capital grows faster than if interest is compounded only once per year.
- ☐ ... the capital grows more slowly than if interest is compounded only once per year.

c) If an initial capital of 1000 CHF grows to 1100 CHF in one year and interest is compounded semiannually ...

- ☐ ... the effective interest rate is less than 10%.
- ☐ ... the effective interest rate is greater than 10%.
- ☐ ... the nominal interest rate is less than 10%.
- ☐ ... the nominal interest rate is greater than 10%.

Answers

11.1 a) i) $C_n = C_0 \left(1+\frac{r}{m}\right)^{mn}$

$C_1 = 1000 \left(1+\frac{0.1}{4}\right)^{4\cdot 1}$ CHF = 1103.81 CHF (rounded)

$C_2 = 1000 \left(1+\frac{0.1}{4}\right)^{4\cdot 2}$ CHF = 1218.40 CHF (rounded)

$C_3 = 1000 \left(1+\frac{0.1}{4}\right)^{4\cdot 3}$ CHF = 1344.89 CHF (rounded)

ii) $r^* = \left(1+\frac{r}{m}\right)^{m} - 1 = \left(1+\frac{0.1}{4}\right)^{4} - 1 = 0.1038 = 10.38\%$ (rounded)

b) i) $C_n = C_0 \left(1+\frac{r}{m}\right)^{mn}$

$C_1 = 1000 \left(1+\frac{0.1}{12}\right)^{12\cdot 1}$ CHF = 1104.71 CHF (rounded)

$C_2 = 1000 \left(1+\frac{0.1}{12}\right)^{12\cdot 2}$ CHF = 1220.39 CHF (rounded)

$C_3 = 1000 \left(1+\frac{0.1}{12}\right)^{12\cdot 3}$ CHF = 1348.18 CHF (rounded)

ii) $r^* = \left(1+\frac{r}{m}\right)^{m} - 1 = \left(1+\frac{0.1}{12}\right)^{12} - 1 = 0.1047 = 10.47\%$ (rounded)

11.2 $r^* = \left(1+\frac{r}{m}\right)^{m} - 1$ $r = 6\% = 0.06$

a) $m = 1$ $r^* = 6\%$

b) $m = 2$ $r^* = 6.09\%$

c) $m = 4$ $r^* = 6.136\%$ (rounded)

d) $m = 12$ $r^* = 6.168\%$ (rounded)

e) $m = 360$ $r^* = 6.183\%$ (rounded)

11.3 $C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$ where C_0 = \$3200, r = 8%, m = 4, n = 5

$\Rightarrow$ C_5 = \$4755.03 (rounded)

11.4 Interest = $C_n - C_0$

$C_n = C_0 \left(1 + \frac{r}{m}\right)^{mn}$ where C_0 = \$10'000, r = 9%, m = 12, n = 3

$\Rightarrow$ $C_n - C_0$ = \$3086.45 (rounded)

11.5 $C_0 = \frac{C_n}{\left(1+\frac{r}{m}\right)^{mn}}$ where C_n = \$40'000, r = 10%, m = 12, n = 18

$\Rightarrow$ C_0 = \$6661.46 (rounded)

11.6 a) $r = m\left(\sqrt[mn]{\frac{C_n}{C_0}} - 1\right)$ where C_0 = \$1000, C_n = \$1500, m = 4, n = 10

$\Rightarrow$ r = 4.08% (rounded)

b) $r^* = \left(1 + \frac{r}{m}\right)^{m} - 1$

$\Rightarrow$ r^* = 4.14% (rounded)

11.7 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where $\frac{C_n}{C_0} = 2$, r = 6%, m = 12, a := 10 (any $a \in \mathbb{R}^+\backslash\{1\}$ would be possible)

$\Rightarrow$ n = 11.58...
$\Rightarrow$ mn = 138.98... $\rightarrow$ 139 months = 11 years 7 months

11.8 A r*(A) = 8.5%

B $r^*(B) = \left(1 + \frac{r}{m}\right)^m - 1$ where r = 8%, m = 12

$\Rightarrow$ r*(B) = 8.3%

$\Rightarrow$ r*(A) > r*(B), i.e. offer A is better than offer B

11.9 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where C_0 = 1000 CHF, C_n = 1250 CHF, r = 2.5%, m = 360, a := 10

$\Rightarrow$ n = 8.92... $\rightarrow$ 9 years

11.10 $r = m\left(\sqrt[mn]{\frac{C_n}{C_0}} - 1\right)$ where C_0 = \$20'000, C_n = \$26'425.82, m = 4, n = 7

$\Rightarrow$ r = 4%

11.11 $n = \frac{\log_a\left(\frac{C_n}{C_0}\right)}{m \cdot \log_a\left(1 + \frac{r}{m}\right)}$ where C_0 = \$10'000, C_n = \$15'000, r = 8%, m = 4, a :=10

$\Rightarrow$ n = 5.11...
$\Rightarrow$ mn = 20.47... $\rightarrow$ 21 quarters = 5 years 3 months

11.12 a) 2nd statement

b) 3rd statement

c) 3rd statement