

Exercises 5 Linear function and equations Linear equations

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to treat applied tasks in economics by means of linear equations.

Problems

5.1 Determine the solution sets of the following equations:

- a) $19x - 32 + 17x = 18x - 30 + 16x - 4$
- b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$
- c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$
- d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

5.2 Determine the solution sets of the following equations:

- a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$
- b) $184 - 6(x - 24) = 214 - 3(2x - 38)$
- c) $(x + 3)(x - 5) = (x - 3)^2$
- d) $(x - 5)(x - 2) = (x - 4)(x - 3)$
- e) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

5.3 Determine the solution sets of the following equations:

- a) $\frac{x+3}{5} = \frac{2x-8}{3}$
- b) $\frac{x+3}{4} + \frac{1-3x}{7} = 0$
- c) $\frac{2}{x-1} = \frac{1}{x-2}$
- d) $\frac{x}{x-1} = \frac{x-1}{x-2}$

5.4 The equations below are equations in the variable x . Furthermore, the equations contain real parameters a and b . Therefore, the solution sets of the equations depend on the values of these parameters.

Solve the equations for x , and determine the solution sets.

Take into account that the parameters a and b can be any real numbers.

a) $x(a - 3) = a$

Hint:

- You may want to divide both sides of the equation by $a - 3$. However, this is not allowed if $a - 3 = 0$, i.e. if $a = 3$, as dividing by 0 is not defined.

b) $(x + 1)(b - 2) = 2bx$

c) $a(1 + x) - 1 = x(2a - 1)$

d) $(a - b)x = a$

- 5.5 The graph of a linear function f with slope a contains the point P . Find the equation of the linear function.
- a) $a = -5$ $P(5|-3)$
 - b) $a = 2$ $P(3|0)$
 - c) $a = 0$ $P(2|3)$
- 5.6 Alps Bikes uses the formula $B(t) = -400t + 5000$ to find the book value $B(t)$, in Swiss francs, of a mountain bike t years after its purchase.
- a) What do the numbers -400 and 5000 signify?
 - b) How long will it take the mountain bike to depreciate completely?
- 5.7 Two items A and B depreciate linearly:
- | | |
|--------|--------------------------------------------------------|
| Item A | original value = 200 CHF
depreciation = 16 CHF/year |
| Item B | original value = 240 CHF
depreciation = 32 CHF/year |
- a) How long will it take the two items to depreciate completely?
 - b) Determine the point in time where both items have the same value.
- Hint:
- Think of the temporal development of the values as linear functions.
- 5.8 Simple interest at an unknown rate is paid on an initial bank balance of 5000 CHF. The balance after five years is 5625 CHF.
- a) Determine the interest rate.
 - b) How long would it take the balance to reach 7000 CHF?

Answers

- 5.1 a) $S = \{-1\}$
 b) $S = \{13\}$
 c) $S = \mathbb{R}$
 d) $S = \{ \}$
- 5.2 a) $S = \left\{ \frac{11}{2} \right\}$
 b) $S = \mathbb{R}$
 c) $S = \{6\}$
 d) $S = \{ \}$
 e) $S = \{0\}$
- 5.3 a) $S = \{7\}$
 b) $S = \{5\}$
 c) $S = \{3\}$
 d) $S = \{ \}$
- 5.4 a) if $a = 3$: no solution $\Rightarrow S = \{ \}$
 if $a \neq 3$: $x = \frac{a}{a-3}$ $\Rightarrow S = \left\{ \frac{a}{a-3} \right\}$
- b) if $b = -2$: no solution $\Rightarrow S = \{ \}$
 if $b \neq -2$: $x = \frac{b-2}{b+2}$ $\Rightarrow S = \left\{ \frac{b-2}{b+2} \right\}$
- c) if $a = 1$: $x \in \mathbb{R}$ $\Rightarrow S = \mathbb{R}$
 if $a \neq 1$: $x = 1$ $\Rightarrow S = \{1\}$
- d) if $a = b = 0$: $x \in \mathbb{R}$ $\Rightarrow S = \mathbb{R}$
 if $a = b \neq 0$: no solution $\Rightarrow S = \{ \}$
 if $a \neq b$: $x = \frac{a}{a-b}$ $\Rightarrow S = \left\{ \frac{a}{a-b} \right\}$
- 5.5 a) $y = f(x) = -5x + 22$
 Hints:
 - The equation of a linear function is $y = f(x) = ax + b$
 - If $P(5|-3)$ is a point of the graph of the linear function, its coordinates must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$
- b) $y = f(x) = 2x - 6$
 c) $y = f(x) = 3$
- 5.6 a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year.
 The number 5000 indicates that the original value of the mountain bike was 5000 CHF.
- b) 12.5 years
 Hint:
 - Complete depreciation at time t means $B(t) = 0$.

- 5.7 a) item A: 12.5 years
 item B: 7.5 years
- b) $t = 2.5$ years (if the original value is meant to be at $t = 0$ years)
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- 5.8 a) slope of the linear function: $a = 125$
 interest rate $r = 2.5\%$
- b) 16 years