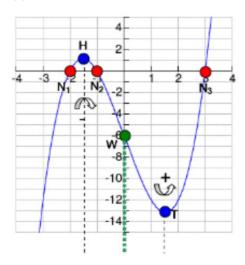
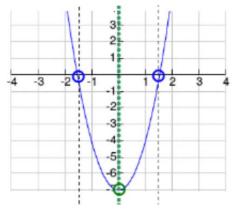
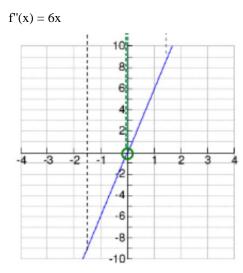
# Increasing/decreasing, concavity

Ex.:  $f(x) = x^3 - 7x - 6$ 









### Increasing/decreasing

The function f is **increasing** at  $x = x_0$ , if the **first derivative** is **positive**, i.e.  $f'(x_0) > 0$ .

The function f is decreasing at  $x = x_0$ , if the first derivative is negative, i.e.  $f'(x_0) < 0$ .

### Concavity

The graph of the function f is concave up at  $x = x_0$ , if the second derivative is positive, i.e.  $f''(x_0) > 0$ .

The graph of the function f is concave down at  $x = x_0$ , if the second derivative is negative, i.e.  $f''(x_0) < 0$ .

#### Relative maxima/minima

The function f has a **relative maximum** at  $x = x_0$ , if the tangent to the graph of f at  $x = x_0$  is horizontal and if the graph of f is concave down at  $x = x_0$ , i.e.  $f'(x_0) = 0$  and  $f''(x_0) < 0$ .

The function f has a **relative minimum** at  $x = x_0$ , if the tangent to the graph of f at  $x = x_0$  is horizontal and if the graph of f is concave up at  $x = x_0$ , i.e.  $f'(x_0) = 0$  and  $f''(x_0) > 0$ .

## Absolute maximum/minimum

The **absolute maximum/minimum** of a continuous function f is either a relative maximum/minimum or the value of f at one of the endpoints of the domain.

#### **Points of inflection**

The function f has a **point of inflection** at  $x = x_0$ , if the graph of f changes its concavity from concave up to concave down (or vice versa) at  $x = x_0$ , i.e. if  $f''(x_0) = 0$  and  $f'''(x_0) \neq 0$ .

Ex.: 
$$f(x) = x^3 - 7x - 6$$
 (see page 1)   
 $\Rightarrow f'(x) = 3x^2 - 7$   
 $\Rightarrow f''(x) = 6x$   
 $\Rightarrow f'''(x) = 6$ 

Relative maxima/minima

$$f'(x) = 0 \text{ at } x_1 = \sqrt{\frac{7}{3}} = 1.52... \text{ and } x_2 = -\sqrt{\frac{7}{3}} = -1.52...$$
  
$$f''(x_1) = 6 \cdot \sqrt{\frac{7}{3}} = 9.16... > 0 \qquad \Rightarrow \text{ relative minimum at } x_1 = \sqrt{\frac{7}{3}}$$
  
$$f''(x_2) = -6 \cdot \sqrt{\frac{7}{3}} = -9.16... < 0 \qquad \Rightarrow \text{ relative maximum at } x_2 = -\sqrt{\frac{7}{3}}$$

Absolute maximum/minimum

Ex.: 
$$D = [0,4]$$
  $\Rightarrow$  absolute maximum at  $x = 4$  (endpoint of domain)  
 $\Rightarrow$  absolute minimum at  $x = x_1 = \sqrt{\frac{7}{3}}$  (relative minimum)

Ex.: 
$$D = [-4,3]$$
  $\Rightarrow$  absolute maximum at  $x = x_2 = -\sqrt{\frac{7}{3}}$  (relative maximum)  
 $\Rightarrow$  absolute minimum at  $x = -4$  (endpoint of domain)

Points of inflection

$$f''(x) = 0 \text{ at } x_3 = 0$$
  
$$f'''(x_3) = 6 \neq 0 \qquad \Rightarrow \text{ point of inflection at } x_3 = 0$$

# **Financial mathematics**

Ex.:

Marginal cost/revenue/profit function = first derivative of the cost/revenue/profit function

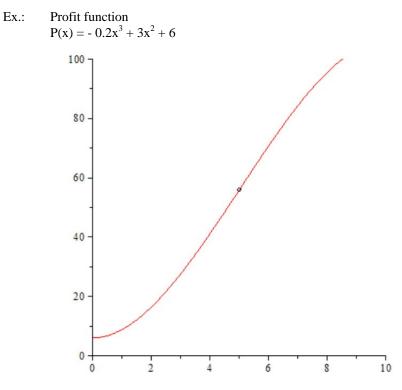
Cost function ⇒ Marginal cost function	$C(x) = 120x + x^{2}$ C'(x) = 120 + 2x
Revenue function $\Rightarrow$ Marginal revenue function	$R(x) = 168x - 0.2x^2$ R'(x) = 168 - 0.4x
Profit function $\Rightarrow$ Marginal profit function	$P(x) = R(x) - C(x) = 48x - 1.2x^2$ P'(x) = 48 - 2.4x

# Average cost/revenue/profit function

Averag	e cost function	$\overline{C}(x) := \frac{C(x)}{x}$	where $C(x) = cost$ function
Ex.:	Cost function ⇒ Average cost function	$C(x) = 3x^2 + 4x$ $\overline{C}(x) = 3x + 4 + 4x$	$+2\frac{2}{x}$
Averag	e revenue function	$\overline{\mathbf{R}}(\mathbf{x}) := \frac{\mathbf{R}(\mathbf{x})}{\mathbf{x}}$	where $R(x)$ = revenue function
Average	e profit function	$\overline{P}(x) := \frac{P(x)}{x}$	where $P(x) = profit$ function

# Point of diminishing returns

Point of diminishing returns = point of inflection on the graph



Point of diminishing returns: (5|56)