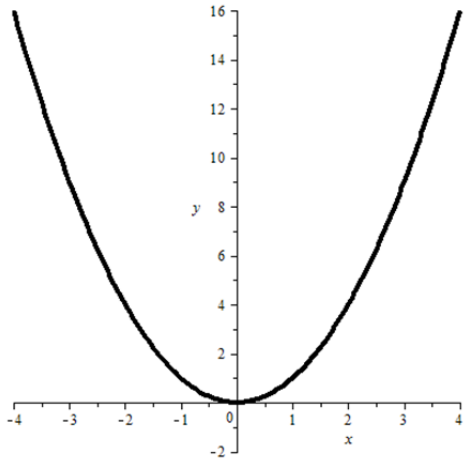
# Quadratic function

## Definition

| | | |
|---|---|---|
| f: D $\rightarrow$ $\mathbb{R}$ | | (D $\subseteq \mathbb{R}$) |
| x $\rightarrow$ | $y = f(x) = ax^2 + bx + c$ general form | ($a \in \mathbb{R}\backslash\{0\}$, $b \in \mathbb{R}$, $c \in \mathbb{R}$) |
| | $y = f(x) = a(x - u)^2 + v$ vertex form | ($a \in \mathbb{R}\backslash\{0\}$, $u \in \mathbb{R}$, $v \in \mathbb{R}$) |

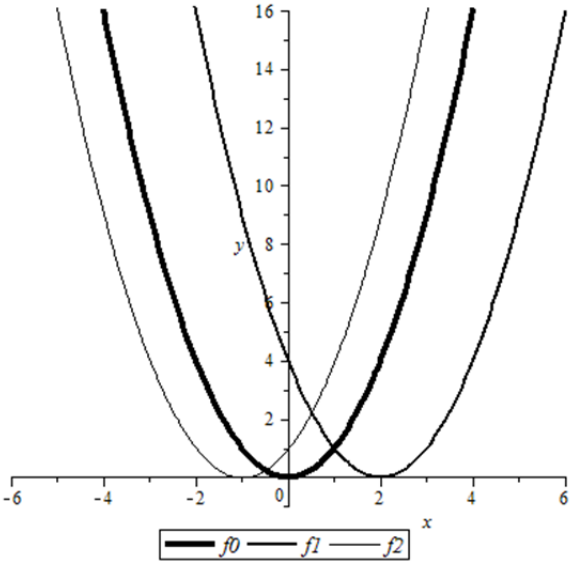
## Graph

1. $y = f(x) = x^2$ ($a = 1$, $u = 0$, $v = 0$)



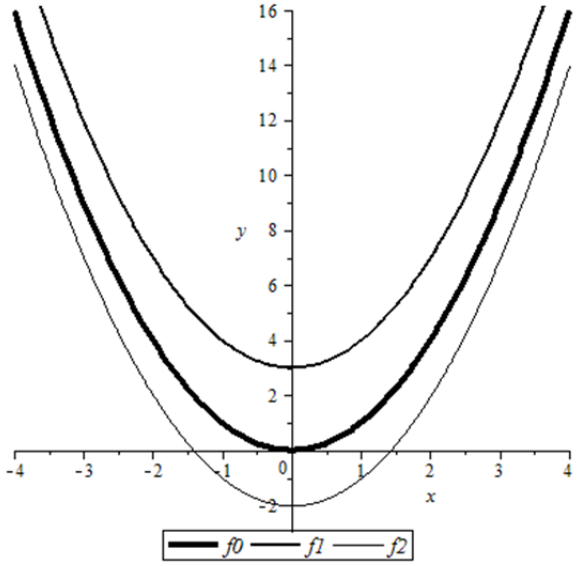

2. Parameter u ($a = 1$, $v = 0$)

$y = f_0(x) = x^2$ ($u = 0$)
$y = f_1(x) = (x - 2)^2$ ($u = 2$)
$y = f_2(x) = (x + 1)^2$ ($u = -1$)



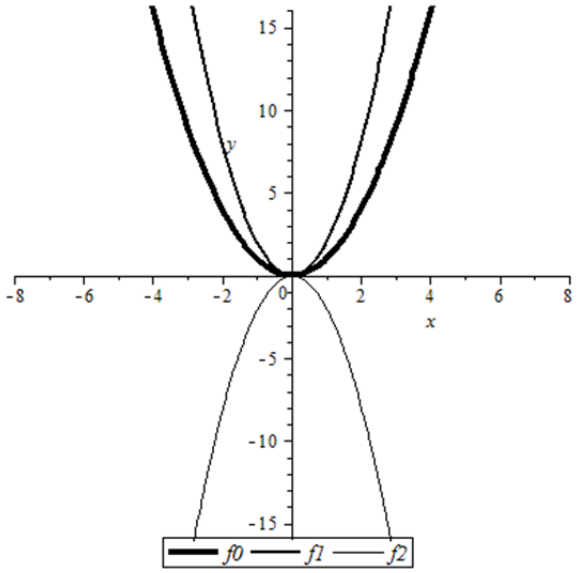
3. Parameter v (a = 1, u = 0)

$y = f_0(x) = x^2$ (v = 0)
$y = f_1(x) = x^2 + 3$ (v = 3)
$y = f_2(x) = x^2 - 2$ (v = -2)

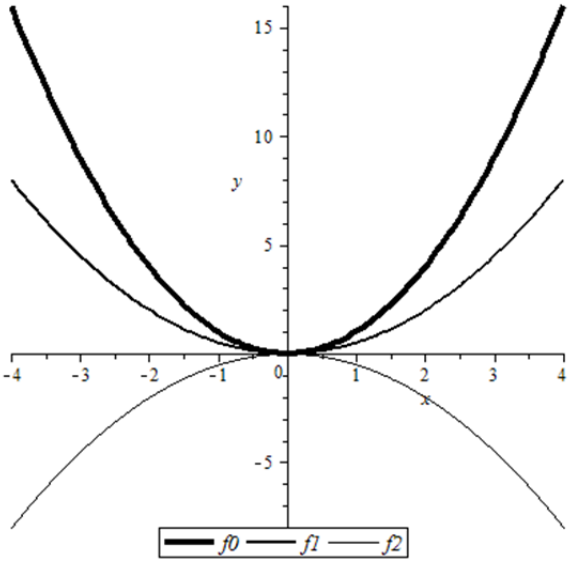



4. Parameter a (u = 0, v = 0)

$y = f_0(x) = x^2$ (a = 1)
$y = f_1(x) = 2x^2$ (a = 2)
$y = f_2(x) = -2x^2$ (a = -2)



$y = f_0(x) = x^2$ $(a = 1)$
$y = f_1(x) = \frac{1}{2}x^2$ $\left(a = \frac{1}{2}\right)$
$y = f_2(x) = -\frac{1}{2}x^2$ $\left(a = -\frac{1}{2}\right)$

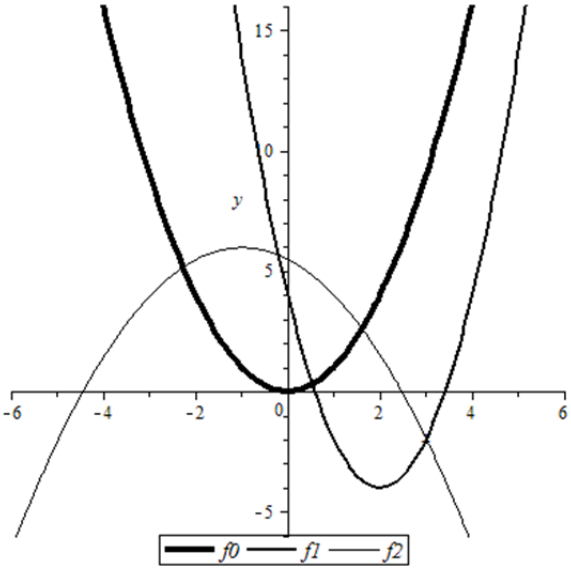



5. The **graph** of a quadratic function is a **parabola**.

The paramter **a** determines the **shape** of the parabola, and whether the parabola opens upwards or downwards.

The parameters **u** and **v** determine the **position** of the parabola. They are the coordinates of the **vertex** V of the parabola: V(u|v)

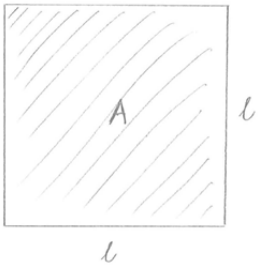
| | | |
|---|---|---|
| $y = f_0(x) = x^2$ | $(a = 1, u = 0, v = 0)$ | V(0\|0) |
| $y = f_1(x) = 2(x-2)^2 - 4$ | $(a = 2, u = 2, v = -4)$ | V(2\|-4) |
| $y = f_2(x) = -\frac{1}{2}(x+1)^2 + 6$ | $\left(a = -\frac{1}{2}, u = -1, v = 6\right)$ | V(-1\|6) |



## Examples

1. Nature/Physics: Trajectories of water in a fountain

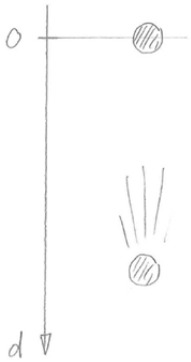
2. Geometry: Square




Area A for side length $l$: $A = l^2$

f: $\mathbb{R}^+ \to \mathbb{R}$
$l \to A = f(l) = l^2$ quadratic function

3. Physics: Free fall




Distance d after time t: $d = \frac{1}{2}gt^2$ (g = gravity field strength)

f: $\mathbb{R} \to \mathbb{R}$
$t \to d = f(t) = \frac{1}{2}gt^2$ quadratic function

4. Economics: Supply, Demand