

Exercises 3 **Function** **Domain, codomain, range, graph**

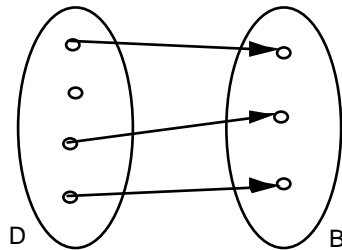
Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

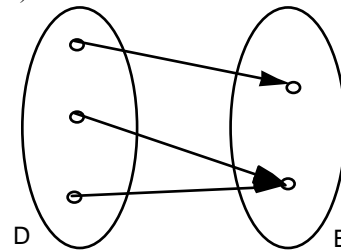
Problems

3.1 Which of the following relations are functions? Explain your answer.

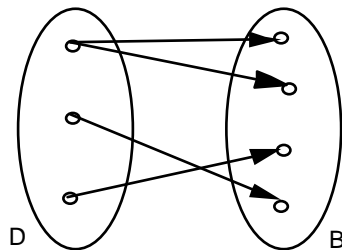
a)



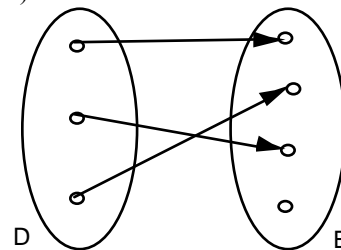
b)



c)



d)



- e) D = set of all the modules of the HTW Tourism bachelor programme
 B = set of all the HTW lecturers
 $f: D \rightarrow B, m \rightarrow l = f(m) = \text{lecturer of } m$
- f) $D = \{1983, 1984, \dots, 1992, 1993\}$
 B = set of all the human beings aged between 20 and 30
 $f: D \rightarrow B, y \rightarrow p = f(y) = \text{person who was born in the year } y$
- g) D = set of all the human beings aged between 20 and 30
 $B = \{1983, 1984, \dots, 1992, 1993\}$
 $f: D \rightarrow B, p \rightarrow y = f(p) = \text{year of birth of person } p$
- h) $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow y = f(x) = x^2$
- i) $f: \mathbb{R}^+ \rightarrow \mathbb{R}, x \rightarrow y = f(x) = \text{number whose square is } x$
 Notice:
 - \mathbb{R}^+ is the set of all positive real numbers, i.e. $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$.
- j) $f: \mathbb{R} \rightarrow \mathbb{R}, t \rightarrow b = f(t) = \text{bank account balance at time } t$

3.2 Determine the range E of the functions below:

- a) $D = \{\text{January, February, March, ..., December}\}$
 $B = \{A, B, C, ..., Z\}$
 $f: D \rightarrow B, m \rightarrow l = f(m) = \text{initial letter of } m$
- b) $D = \text{set of all the neighbouring countries of Switzerland}$
 $B = \text{set of all the European cities}$
 $c: D \rightarrow B, x \rightarrow y = c(x) = \text{capital of neighbouring country } x$
- c) function f in problem 3.1 g)
- d) function f in problem 3.1 h)

3.3 a) $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow f(x) = x^3 - x$

Determine the following values:

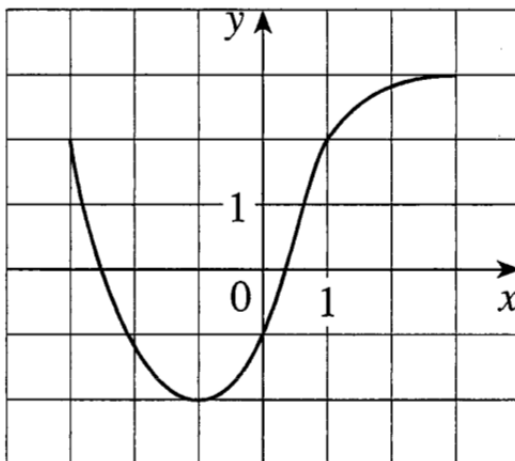
- i) $f(1)$ ii) $f(-2)$ iii) $f(a)$
- iv) $f(b^2)$ v) $f(a - b)$ vi) $f(x^3 - x)$

b) $g: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}, x \rightarrow g(x) = \frac{x^2}{x+1}$

Determine the following values:

- i) $g(2)$ ii) $g(-3)$ iii) $g(a)$
- iv) $g(b^2)$ v) $g(a - b)$ vi) $g\left(\frac{x^2}{x+1}\right)$

3.4 The graph of a function f is given as follows:



- a) State the value of $f(-1)$.
- b) Estimate the value of $f(2)$.
- c) For what values of x is $f(x) = 2$?
- d) Estimate the values of x such that $f(x) = 0$.
- e) State the domain D of f .
- f) State the range E of f .

Answers

- 3.1 a) no function b) function
 c) no function d) function
 e) no function f) no function
 g) function h) function
 i) no function j) function

- 3.2 a) $E = \{A, D, F, J, M, N, O, S\}$
 b) $E = \{\text{Berlin, Vienna, Vaduz, Rome, Paris}\}$
 c) $E = B$
 d) $E = \mathbb{R}_0^+$

Notice:

- \mathbb{R}_0^+ is the set of all positive real numbers, including zero, i.e. $\mathbb{R}_0^+ = \{x: x \in \mathbb{R} \text{ and } x \geq 0\}$.

- 3.3 a) i) $f(1) = 1^3 - 1 = 0$
 ii) $f(-2) = (-2)^3 - (-2) = -6$
 iii) $f(a) = a^3 - a$
 iv) $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$
 v) $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$
 vi) $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$

- b) i) $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$
 ii) $g(-3) = \frac{(-3)^2}{-3+1} = -\frac{9}{2}$
 iii) $g(a) = \frac{a^2}{a+1}$
 iv) $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$
 v) $g(a - b) = \frac{(a-b)^2}{(a-b)+1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$
 vi) $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right)+1} = \frac{x^4}{x^3 + 2x^2 + 2x + 1}$

- 3.4 a) $f(-1) = -2$
 b) $f(2) \approx 2.8$
 c) $x_1 = -3, x_2 = 1$
 d) $x_1 \approx -2.5, x_2 \approx 0.3$
 e) $D = \{x: x \in \mathbb{R} \text{ and } -3 \leq x \leq 3\} = [-3, 3]$
 f) $E = \{y: y \in \mathbb{R} \text{ and } -2 \leq y \leq 3\} = [-2, 3]$