

Indefinite integral

Ex.: Financial mathematics

Given the marginal cost function $C'(x)$ for the production of a commodity:

$$C'(x) = 3x + 50$$

What is the cost function $C(x)$?

$$C(x) = \dots ?$$

General problem

Given a function f . What function F is such that $F' = f$?

Ex.: $f(x) = 2x$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = x^2 && \text{as } F_1'(x) = 2x = f(x) \\ & F_2(x) = x^2 + 1 && \text{as } F_2'(x) = 2x + 0 = 2x = f(x) \\ & F_3(x) = x^2 - 4 && \text{as } F_3'(x) = 2x + 0 = 2x = f(x) \\ & \dots && \\ & F(x) = x^2 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 2x + 0 = 2x = f(x) \end{aligned}$$

$$f(x) = 8x^3$$

$$\begin{aligned} \Rightarrow \quad & F_1(x) = 2x^4 && \text{as } F_1'(x) = 8x^3 = f(x) \\ & F_2(x) = 2x^4 + 5 && \text{as } F_2'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & F_3(x) = 2x^4 - 11 && \text{as } F_3'(x) = 8x^3 + 0 = 8x^3 = f(x) \\ & \dots && \\ & F(x) = 2x^4 + C \quad (C \in \mathbb{R}) && \text{as } F'(x) = 8x^3 + 0 = 8x^3 = f(x) \end{aligned}$$

Definitions

$F(x)$ is called an **antiderivative** of $f(x)$ if its derivative $F'(x)$ is equal to $f(x)$, i.e. $F'(x) = f(x)$.

The set of all antiderivatives of the function $f(x)$ is called the **indefinite integral** of $f(x)$, denoted $\int f(x) \, dx$.

$$\int f(x) \, dx = F(x) + C$$

C ($C \in \mathbb{R}$) is called the **integration constant**.

Ex.: $f(x) = 8x^3$

The functions $F_1(x) = 2x^4$, $F_2(x) = 2x^4 + 5$, $F_3(x) = 2x^4 - 11$, ... are antiderivatives of $f(x)$.

We therefore write $\int f(x) \, dx = \int 8x^3 \, dx = 2x^4 + C$

$$f(x) = 12x^2$$

$$\int f(x) \, dx = \int 12x^2 \, dx = 4x^3 + C$$

$$\int 2x \, dx = x^2 + C$$

$$\int 3 e^{3x} \, dx = e^{3x} + C$$