

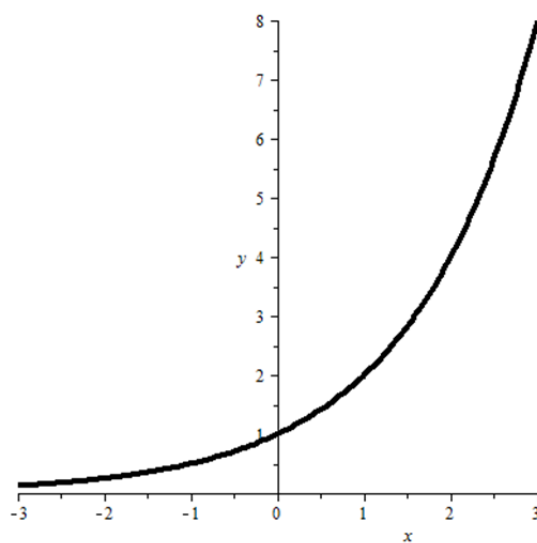
# Exponential function

## Definition

f: $D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \rightarrow y = f(x) = c \cdot a^x$	$(a \in \mathbb{R}^+ \setminus \{1\}, c \in \mathbb{R} \setminus \{0\})$
$a > 1$ : exponential <b>growth</b>	
$a < 1$ : exponential <b>decay</b>	

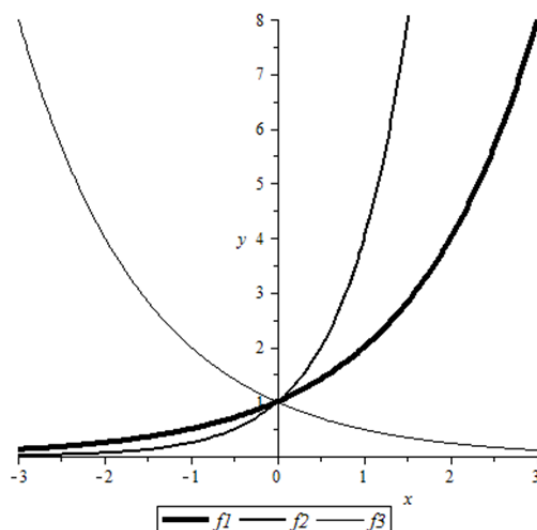
## Graph

1.  $y = f(x) = 2^x$   $(c = 1, a = 2)$



2. Parameter a

$$\begin{aligned} y = f_1(x) &= 2^x & (c = 1, a = 2) \\ y = f_2(x) &= 4^x & (c = 1, a = 4) \\ y = f_3(x) &= \left(\frac{1}{2}\right)^x & \left(c = 1, a = \frac{1}{2}\right) \end{aligned}$$

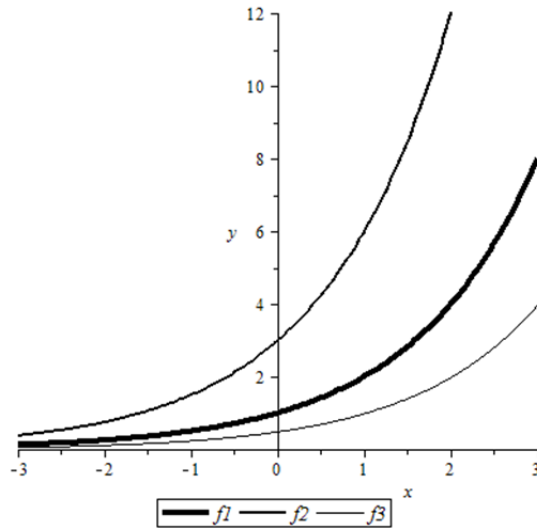


3. Parameter c

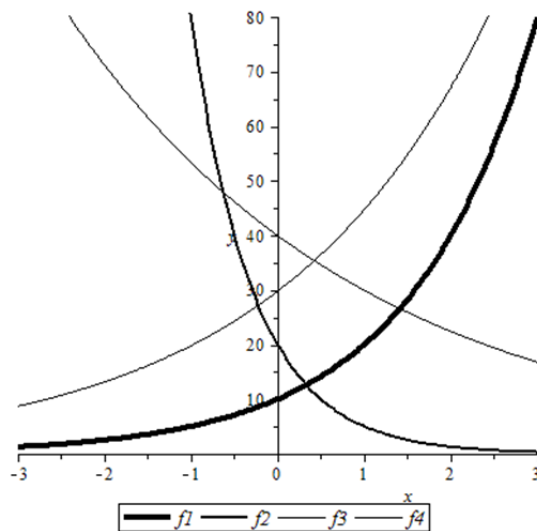
$$y = f_1(x) = 2^x \quad (a = 1, c = 1)$$

$$y = f_2(x) = 3 \cdot 2^x \quad (a = 1, c = 3)$$

$$y = f_3(x) = \frac{1}{2} \cdot 2^x \quad \left(a = 1, c = \frac{1}{2}\right)$$



- 4.
- $$y = f_1(x) = 10 \cdot 2^x \quad (c = 10, a = 2)$$
- $$y = f_2(x) = 20 \cdot 0.25^x \quad (c = 20, a = 0.25)$$
- $$y = f_3(x) = 40 \cdot 0.75^x \quad (c = 40, a = 0.75)$$
- $$y = f_4(x) = 30 \cdot 1.5^x \quad (c = 30, a = 1.5)$$



## Examples

1. Compound interest (exponential **growth**)

$$C_n = C_0 \cdot q^n$$

$C_0$  = initial capital

$C_n$  = capital after n compounding periods

n = number of compounding periods (typically: 1 compounding period = 1 year)

q = growth factor =  $1 + r$  ( $q > 1$ )

r = interest rate per compounding period

$$\text{Ex.: } C_0 := 1000, r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$$

2. Consumer price index (exponential **decay**)

$$P(t) = P_0 \cdot q^t$$

$P_0$  = initial purchasing power

$P(t)$  = purchasing power at time t (typically: t in years)

q = decay factor ( $q < 1$ )

$$\text{Ex.: } P_0 := 100, q := 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$$