Exercises 17 Definite integral
Definite integral, area under a curve, consumer's/producer's surplus

Objectives

- be able to determine the definite integral of a constant/basic power/basic exponential function.
- be able to determine the area between the graph of a basic power function and the abscissa.
- be able to determine the consumer's/producer's surplus if the demand and supply functions are basic power functions.

Problems

17.1 Calculate the definite integrals below:

a) $\int_3^4 (2x - 5)\, dx$ b) $\int_0^1 (x^3 + 2x)\, dx$ c) $\int_{-5}^{-3} \left(\frac{x^2}{2} - 4\right) dx$

d) $\int_2^4 \left(x^3 - \frac{x^2}{2} + 3x - 4\right) dx$ e) $\int_{-2}^{2} \left(2x^2 - \frac{x^4}{8}\right) dx$ f) $\int_{-1}^{1} e^x\, dx$

17.2 Determine the area between the graph of the function and the x-axis on the interval where the graph of f is above the x-axis, i.e. where $f(x) \geq 0$.

a) $f(x) = - x^2 + 1$ b) $f(x) = x^3 - x^2 - 2x$

17.3 The demand function for a product is $p = f(x) = 100 - 4x^2$.
If the equilibrium quantity is 4 units, what is the consumer's surplus?

17.4 The demand function for a product is $p = f(x) = 34 - x^2$.
If the equilibrium price is \$9, what is the consumer's surplus?

17.5 The demand function for a certain product is
$p = f(x) = 81 - x^2$
and the supply function is
$p = g(x) = x^2 + 4x + 11$.

Find the equilibrium point and the consumer's surplus there.

17.6 Suppose that the supply function for a good is $p = g(x) = 4x^2 + 2x + 2$.
If the equilibrium price is \$422, what is the producer's surplus?

17.7 Find the producer's surplus for a product if its demand function is
$p = f(x) = 81 - x^2$
and its supply function is
$p = g(x) = x^2 + 4x + 11$

17.8 The demand function for a certain product is
$p = f(x) = 144 - 2x^2$
and the supply function is
$p = g(x) = x^2 + 33x + 48$

Find the producer's surplus at the equilibrium point.

Answers

17.1 a) $\int_3^4 (2x - 5)\,dx = [x^2 - 5x]_3^4 = (4^2 - 5\cdot 4) - (3^2 - 5\cdot 3) = 2$

b) $\int_0^1 (x^3 + 2x)\,dx = \left[\frac{x^4}{4} + x^2\right]_0^1 = \left(\frac{1^4}{4} + 1^2\right) - \left(\frac{0^4}{4} + 0^2\right) = \frac{5}{4}$

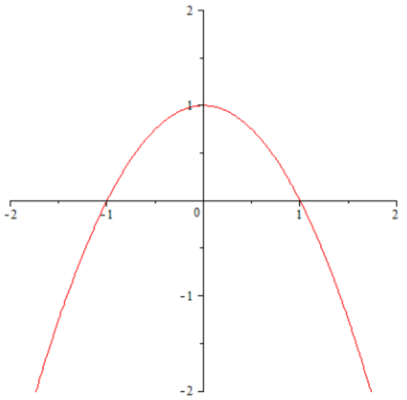
c) $\int_{-5}^{-3} \left(\frac{x^2}{2} - 4\right)dx = \left[\frac{x^3}{6} - 4x\right]_{-5}^{-3} = \left(\frac{(-3)^3}{6} - 4\cdot(-3)\right) - \left(\frac{(-5)^3}{6} - 4\cdot(-5)\right) = \frac{25}{3}$

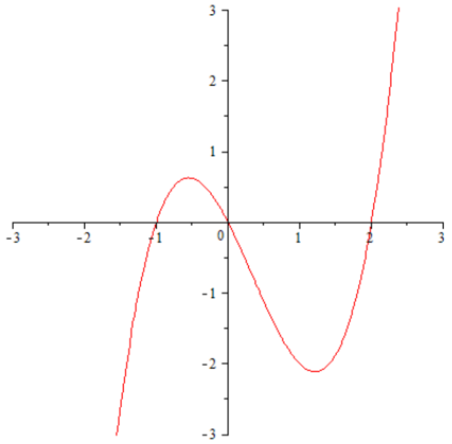
d) $\int_2^4 \left(x^3 - \frac{x^2}{2} + 3x - 4\right)dx = \left[\frac{x^4}{4} - \frac{x^3}{6} + \frac{3x^2}{2} - 4x\right]_2^4 = \left(\frac{4^4}{4} - \frac{4^3}{6} + \frac{3\cdot 4^2}{2} - 4\cdot 4\right) - \left(\frac{2^4}{4} - \frac{2^3}{6} + \frac{3\cdot 2^2}{2} - 4\cdot 2\right) = \frac{182}{3}$

e) $\int_{-2}^2 \left(2x^2 - \frac{x^4}{8}\right)dx = \left[\frac{2x^3}{3} - \frac{x^5}{40}\right]_{-2}^2 = \left(\frac{2\cdot 2^3}{3} - \frac{2^5}{40}\right) - \left(\frac{2\cdot(-2)^3}{3} - \frac{(-2)^5}{40}\right) = \frac{136}{15}$

f) $\int_{-1}^1 e^x\,dx = [e^x]_{-1}^1 = e^1 - e^{-1} = e - \frac{1}{e}$

17.2 a) $A = \int_{-1}^1 (-x^2 + 1)\,dx = \left[-\frac{x^3}{3} + x\right]_{-1}^1 = \frac{4}{3}$ b) $A = \int_{-1}^0 (x^3 - x^2 - 2x)\,dx = \left[\frac{x^4}{4} - \frac{x^3}{3} - x^2\right]_{-1}^0 = \frac{5}{12}$





Hints:
- First, find the positions x where the graph of f intersects the x-axis, i.e where $f(x) = 0$
- Then, find the interval on which the graph of f is above the x-axis, i.e. where $f(x) \geq 0$

17.3 Consumer's surplus CS = \$170.67

17.4 Consumer's surplus CS = \$83.33

17.5 Equilibrium quantity x = 5
Equilibrium price p = \$56
Consumer's surplus CS = \$83.33

17.6 Producer's surplus PS = \$2766.67

17.7 Producer's surplus PS = \$133.33

17.8 Producer's surplus PS = \$103.34