Exercises 6 Linear function and equations
Linear systems of equations

Objectives

- be able to solve a linear system of equations.
- be able to treat applied tasks by means of linear systems of equations.

Problems

6.1 Solve the following systems of equations:

a) $4x + 3y = 14$
$2x - y = 12$

b) $-4a - b = 40$
$a + 5b = 9$

c) $12x + 9y = 15$
$4x + 3y = 5$

d) $a - 4b = 3$
$-5a + 20b = 10$

e) $2p - 6q = 6$
$5p + 3q = 42$

f) $2x + 3y + 5 = 5x + 6y - 1$
$x - 4y - 2 = 2x - 2y$

g) $3(x + 5) = 2(2y - 1)$
$4(3x - 6) = 3(y + 4)$

h) $(x + 5)(y + 1) = (x + 8)(y - 3)$
$(x - 3)(y - 1) = (x - 1)(y + 3)$

i) $2(2a + 3b) = 3(3a - b) + 5$
$4(3a - 4b) = 2(a + b) - 10$

6.2 Find the equation of the linear function whose graph contains the two points P and Q:

a) $P(5|-3)$ $Q(-2|1)$

b) $P(2|-3)$ $Q(-1|-4)$

c) $P(3|-7)$ $Q(3|-9)$

6.3 Find the intersection point of the graphs of the two linear functions f and g:

a) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = -3x + \frac{5}{4}$ $g: \mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = -x - 1$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = 2x + \frac{5}{4}$ $g: \mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = 2x - 1$

6.4 Find out whether the graphs of the linear functions f, g, and h have a point P in common.

a) f: $\mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = x + 1$; g: $\mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = -\frac{x}{2} - 2$; h: $\mathbb{R} \to \mathbb{R}$, $x \to y = h(x) = \frac{5}{3}x + \frac{7}{3}$

b) f: $\mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = \frac{1}{6}x + \frac{3}{2}$; g: $\mathbb{R} \to \mathbb{R}$, $x \to y = g(x) = -\frac{2}{3}x + 2$; h: $\mathbb{R} \to \mathbb{R}$, $x \to y = h(x) = 2x - 3$

6.5 Hotelier A says to hotelier B: "If three quarters of your hotel guests moved to my hotel, I would host 100 guests." Hotelier B replies: "If half of your guests moved to my hotel, I would host 100 guests."

How many guests do A and B host in their hotels?

6.6 The (non-linear) equation $ax^2 + bx = 1$ has the solution set S = {2, 3}, i.e. the equation has the two solutions $x_1 = 2$ and $x_2 = 3$.

Determine the values of the parameters a and b.

6.7 $3000 are awarded to three winners. The first prize is 5/3 of the second one, whereas the second prize is 3/2 of the third one.

Determine the values of the three prizes.

6.8 In a family, the mother is 32 years older than her daughter, whereas the father is 26 years older than his son. In a sum, the mother and her daughter are 10 years older than the father. The difference of the son's and the daughter's ages is twice as much as the difference of the two parents' ages.

Determine the age of each family member.

6.9 Red Tide and Blue Flake are planning new lines of skis.

Red Tide
For the first year, the fixed costs for setting up production are $45'000. The variable costs for producing each pair of skis are estimated at $80, and the selling price will be $255 per pair. It is projected that 3000 pairs will sell the first year.

Blue Flake
For the first year, the fixed costs for setting up production are $40'000. The variable costs for producing each pair of skis are estimated at $80, and the selling price will be $250 per pair. It is projected that 3500 pairs will sell the first year.

How many pairs of skis must both Red Tide and Blue Flake sell in order to realise the same profit?
What is the profit?

Answers

6.1 a) $(x, y) = (5, -2)$

b) $(a, b) = (-11, 4)$

c) infinitely many solutions
$S = \{(x, (5-4x)/3) : x \in \mathbb{R}\}$

d) no solution
$S = \{ \}$

e) $(p, q) = (15/2, 3/2)$

f) $(x, y) = (6, -4)$

g) $(x, y) = (5, 8)$

h) $(x, y) = (-2, 7)$

i) infinitely many solutions
$S = \{(a, 5(1+a)/9) : a \in \mathbb{R}\}$

6.2 a) $y = f(x) = -\frac{4}{7}x - \frac{1}{7}$

Hints:
- The equation of a linear function is $y = f(x) = ax + b$
- If P(5|-3) and Q(-2|1) are points of the graph of the linear function, their coordinates must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$ and $1 = f(-2) = a \cdot (-2) + b$
- Solve the following system of equations:

$$-3 = 5a + b$$
$$1 = -2a + b$$

b) $y = f(x) = \frac{1}{3}x - \frac{11}{3}$

c) slope is not defined, therefore no function

6.3 a) P(9/8 | -17/8)

Hint:
- If $P(x_1|y_1)$ is an intersection point of the graphs of two functions f and g, its coordinates x_1 and y_1 must fulfil the two equations $y_1 = f(x_1)$ and $y_1 = g(x_1)$
- Solve the following system of equations:

$$y = -3x + \frac{5}{4}$$
$$y = -x - 1$$

b) no intersection point as graphs are parallel

6.4 a) P(-2|-1)

b) no intersection point P
graphs of f and g intersect at P(3/5 | 8/5), however graph of h does not contain P

6.5 A: 40 guests B: 80 guests

Hints:
- Convert the two statements of the hoteliers into two equations, i.e. into a system of two equations, where the numbers of guests in the two hotels are the variables.
- Solve the system of equations.

6.6 $a = -\frac{1}{6}$ $b = \frac{5}{6}$

6.7 1st prize = \$1500 2nd prize = \$900 3rd prize = \$600

6.8 (f, m, s, d) := (father's age, mother's age, son's age, daughter's age)

2 solutions: $(f, m, s, d)_1 = (54, 48, 28, 16)$
$(f, m, s, d)_2 = (38, 40, 12, 8)$

6.9 Red Tide

Total costs $C_1(x) = 80x + 45'000$
Revenue $R_1(x) = 255x$
Profit $P_1(x) = R_1(x) - C_1(x) = 175x - 45'000$

Blue Flake

Total costs $C_2(x) = 80x + 40'000$
Revenue $R_2(x) = 250x$
Profit $P_2(x) = R_2(x) - C_2(x) = 170x - 40'000$

$P_2(x) = P_1(x)$

$\Rightarrow$ $x = 1000$, $P_1(1000) = P_2(1000) = 130'000$
1000 pairs of skis, profit = \$130'000