

## Exercises 3                      **Function** **Domain, codomain, range, graph**

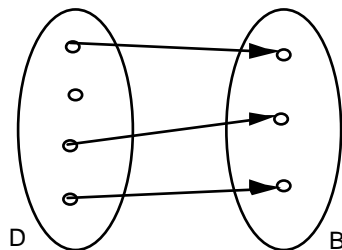
### Objectives

- understand what a function is.
- be able to judge whether a given relation is a function.
- be able to determine the range of a given function.
- be able to determine values of a given function.

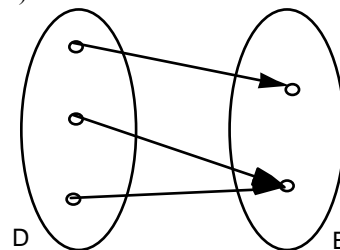
### Problems

3.1 Which of the following relations are functions? Explain your answer.

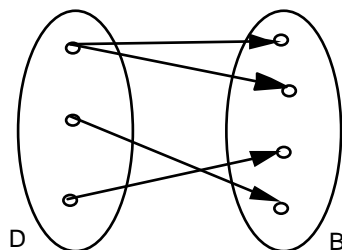
a)



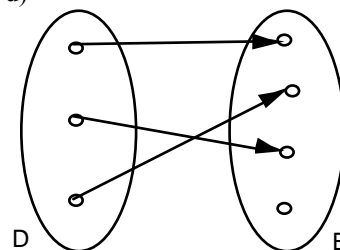
b)



c)



d)



e)      $D =$  set of all the modules of the HTW Tourism bachelor programme  
         $B =$  set of all the HTW lecturers  
         $f: D \rightarrow B, m \rightarrow l = f(m) =$  lecturer of  $m$

f)      $D = \{1982, 1983, \dots, 1991, 1992\}$   
         $B =$  set of all the human beings aged between 20 and 30  
         $f: D \rightarrow B, y \rightarrow p = f(y) =$  person who was born in the year  $y$

g)      $D =$  set of all the human beings aged between 20 and 30  
         $B = \{1982, 1983, \dots, 1991, 1992\}$   
         $f: D \rightarrow B, p \rightarrow y = f(p) =$  year of birth of person  $p$

h)      $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow y = f(x) = x^2$

i)      $f: \mathbb{R}^+ \rightarrow \mathbb{R}, x \rightarrow y = f(x) =$  number whose square is  $x$

Notice:

-  $\mathbb{R}^+$  is the set of all positive real numbers, i.e.  $\mathbb{R}^+ = \{x: x \in \mathbb{R} \text{ and } x > 0\}$ .

j)      $f: \mathbb{R} \rightarrow \mathbb{R}, t \rightarrow b = f(t) =$  bank account balance at time  $t$

3.2 Determine the range E of the functions below:

- a)  $D = \{\text{January, February, March, ..., December}\}$   
 $B = \{A, B, C, ..., Z\}$   
 $f: D \rightarrow B, m \rightarrow l = f(m) = \text{initial letter of } m$
- b)  $D = \text{set of all the neighbouring countries of Switzerland}$   
 $B = \text{set of all the European cities}$   
 $c: D \rightarrow B, x \rightarrow y = c(x) = \text{capital of neighbouring country } x$
- c) function  $f$  in problem 3.1 g)
- d) function  $f$  in problem 3.1 h)

3.3 a)  $f: \mathbb{R} \rightarrow \mathbb{R}, x \rightarrow f(x) = x^3 - x$

Determine the following values:

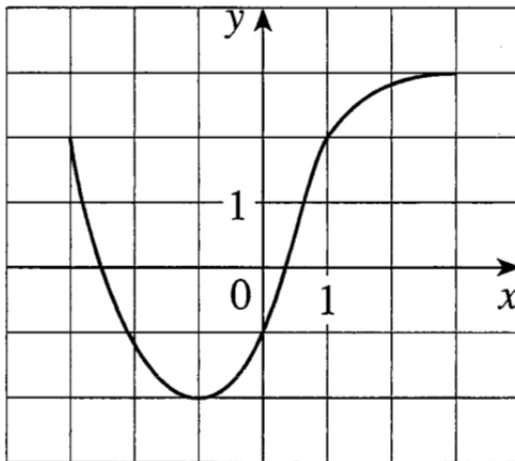
- i)  $f(1)$                       ii)  $f(-2)$                       iii)  $f(a)$
- iv)  $f(b^2)$                       v)  $f(a - b)$                       vi)  $f(x^3 - x)$

b)  $g: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}, x \rightarrow g(x) = \frac{x^2}{x+1}$

Determine the following values:

- i)  $g(2)$                       ii)  $g(-3)$                       iii)  $g(a)$
- iv)  $g(b^2)$                       v)  $g(a - b)$                       vi)  $g\left(\frac{x^2}{x+1}\right)$

3.4 The graph of a function  $f$  is given as follows:



- a) State the value of  $f(-1)$ .
- b) Estimate the value of  $f(2)$ .
- c) For what values of  $x$  is  $f(x) = 2$ ?
- d) Estimate the values of  $x$  such that  $f(x) = 0$ .
- e) State the domain  $D$  of  $f$ .
- f) State the range  $E$  of  $f$ .

**Answers**

- 3.1    a)    no function                      b)    function  
        c)    no function                      d)    function  
        e)    no function                      f)    no function  
        g)    function                            h)    function  
        i)    no function                      j)    function

- 3.2    a)     $E = \{A, D, F, J, M, N, O, S\}$   
        b)     $E = \{\text{Berlin, Vienna, Vaduz, Rome, Paris}\}$   
        c)     $E = B$   
        d)     $E = \mathbb{R}_0^+$

Notice:

-  $\mathbb{R}_0^+$  is the set of all positive real numbers, including zero, i.e.  $\mathbb{R}_0^+ = \{x: x \in \mathbb{R} \text{ and } x \geq 0\}$ .

- 3.3    a)    i)         $f(1) = 1^3 - 1 = 0$   
           ii)         $f(-2) = (-2)^3 - (-2) = -6$   
           iii)         $f(a) = a^3 - a$   
           iv)         $f(b^2) = (b^2)^3 - b^2 = b^6 - b^2$   
           v)         $f(a - b) = (a - b)^3 - (a - b) = a^3 - 3a^2b + 3ab^2 - b^3 - a + b$   
           vi)         $f(x^3 - x) = (x^3 - x)^3 - (x^3 - x) = x^9 - 3x^7 + 3x^5 - 2x^3 + x$

- b)    i)         $g(2) = \frac{2^2}{2+1} = \frac{4}{3}$   
           ii)         $g(-3) = \frac{(-3)^2}{-3+1} = -\frac{9}{2}$   
           iii)         $g(a) = \frac{a^2}{a+1}$   
           iv)         $g(b^2) = \frac{(b^2)^2}{b^2+1} = \frac{b^4}{b^2+1}$   
           v)         $g(a - b) = \frac{(a-b)^2}{(a-b)+1} = \frac{a^2 - 2ab + b^2}{a - b + 1}$   
           vi)         $g\left(\frac{x^2}{x+1}\right) = \frac{\left(\frac{x^2}{x+1}\right)^2}{\left(\frac{x^2}{x+1}\right)+1} = \frac{x^4}{x^3 + 2x^2 + 2x + 1}$

- 3.4    a)     $f(-1) = -2$   
        b)     $f(2) \approx 2.8$   
        c)     $x_1 = -3, x_2 = 1$   
        d)     $x_1 \approx -2.5, x_2 \approx 0.3$   
        e)     $D = \{x: x \in \mathbb{R} \text{ and } -3 \leq x \leq 3\} = [-3, 3]$   
        f)     $E = \{y: y \in \mathbb{R} \text{ and } -2 \leq y \leq 3\} = [-2, 3]$