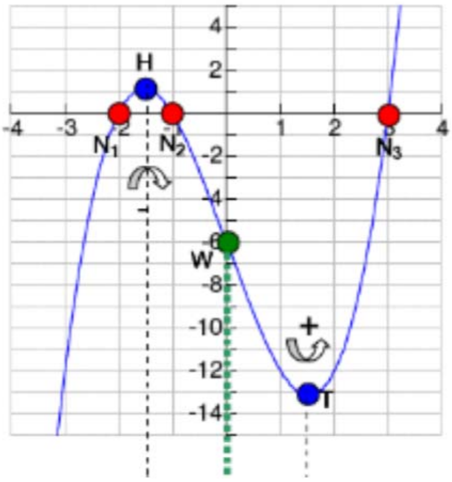
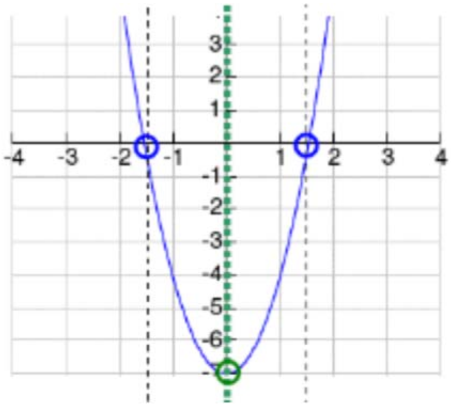
Increasing/decreasing, concavity

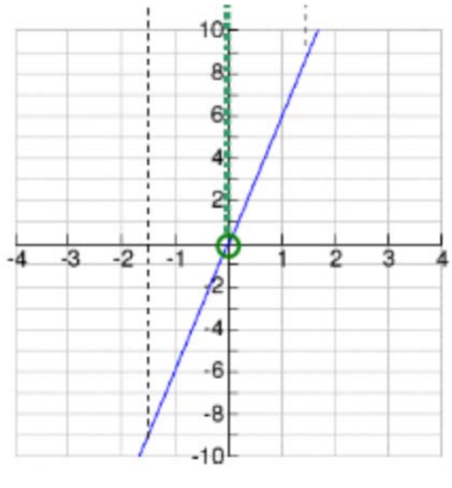
Ex.: $f(x) = x^3 - 7x - 6$



$f'(x) = 3x^2 - 7$



$f''(x) = 6x$



Increasing/decreasing

The function f is **increasing** at $x = x_0$, if the **first derivative** is **positive**, i.e. $f'(x_0) > 0$.

The function f is **decreasing** at $x = x_0$, if the **first derivative** is **negative**, i.e. $f'(x_0) < 0$.

Concavity

The graph of the function f is **concave up** at $x = x_0$, if the **second derivative** is **positive**, i.e. $f''(x_0) > 0$.

The graph of the function f is **concave down** at $x = x_0$, if the **second derivative** is **negative**, i.e. $f''(x_0) < 0$.

Relative maxima/minima

The function f has a **relative maximum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave down at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) < 0$.

The function f has a **relative minimum** at $x = x_0$, if the tangent to the graph of f at $x = x_0$ is horizontal and if the graph of f is concave up at $x = x_0$, i.e. $f'(x_0) = 0$ and $f''(x_0) > 0$.

Absolute maximum/minimum

The **absolute maximum/minimum** of a continuous function f is either a relative maximum/minimum or the value of f at one of the endpoints of the domain.

Points of inflection

The function f has a **point of inflection** at $x = x_0$, if the graph of f changes its concavity from concave up to concave down (or vice versa) at $x = x_0$, i.e. if $f''(x_0) = 0$ and $f'''(x_0) \neq 0$.

Ex.: $f(x) = x^3 - 7x - 6$ (see page 1) $\Rightarrow f'(x) = 3x^2 - 7$

$\Rightarrow f''(x) = 6x$

$\Rightarrow f'''(x) = 6$

Relative maxima/minima

$f'(x) = 0$ at $x_1 = \sqrt{\frac{7}{3}} = 1.52\ldots$ and $x_2 = -\sqrt{\frac{7}{3}} = -1.52\ldots$

$f''(x_1) = 6 \cdot \sqrt{\frac{7}{3}} = 9.16\ldots > 0$ $\Rightarrow$ relative minimum at $x_1 = \sqrt{\frac{7}{3}}$

$f''(x_2) = -6 \cdot \sqrt{\frac{7}{3}} = -9.16\ldots < 0$ $\Rightarrow$ relative maximum at $x_2 = -\sqrt{\frac{7}{3}}$

Absolute maximum/minimum

If domain $D = [0,4]$ $\Rightarrow$ absolute maximum at $x = 4$ (endpoint of domain)

$\Rightarrow$ absolute minimum at $x = x_1 = \sqrt{\frac{7}{3}}$ (relative minimum)

If domain $D = [-4,3]$ $\Rightarrow$ absolute maximum at $x = x_2 = -\sqrt{\frac{7}{3}}$ (relative maximum)

$\Rightarrow$ absolute minimum at $x = -4$ (endpoint of domain)

Points of inflection

$f''(x) = 0$ at $x_3 = 0$

$f'''(x_3) = 6 \neq 0$ $\Rightarrow$ point of inflection at $x_3 = 0$

Financial mathematics

Marginal cost/revenue/profit function = first derivative of the cost/revenue/profit function

Ex.:	Cost function	$C(x) = 120x + x^2$
	⇒ Marginal cost function	$C'(x) = 120 + 2x$
	Revenue function	$R(x) = 168x - 0.2x^2$
	⇒ Marginal revenue function	$R'(x) = 168 - 0.4x$
	Profit function	$P(x) = R(x) - C(x) = 48x -1.2x^2$
	⇒ Marginal profit function	$P'(x) = 48 - 2.4x$

Average cost/revenue/profit function

Average cost function $\overline{C}(x) := \frac{C(x)}{x}$ where C(x) = cost function

Ex.: Cost function $C(x) = 3x^2 + 4x + 2$

⇒ Average cost function $\overline{C}(x) = 3x + 4 + \frac{2}{x}$

Average revenue function $\overline{R}(x) := \frac{R(x)}{x}$ where R(x) = revenue function

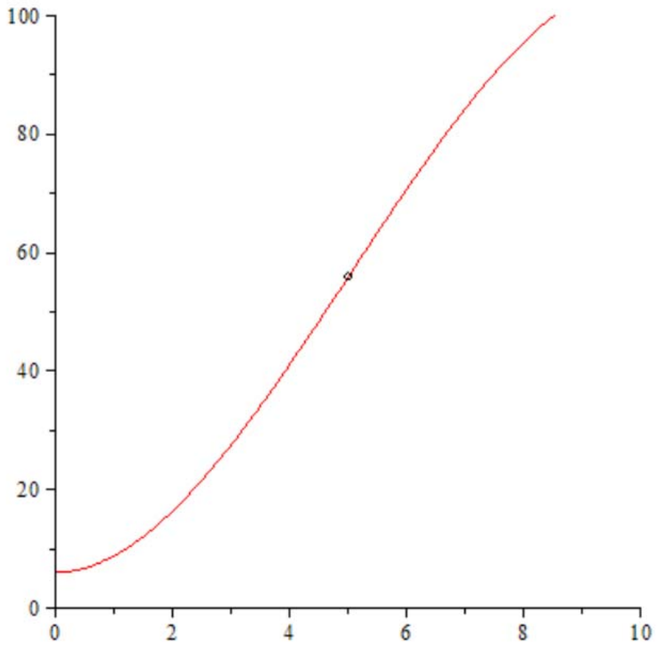
Average profit function $\overline{P}(x) := \frac{P(x)}{x}$ where P(x) = profit function

Point of diminishing returns

Point of diminishing returns = point of inflection on the graph

Ex.: Profit function

$P(x) = - 0.2x^3 + 3x^2 + 6$



Point of diminishing returns: (5|56)