

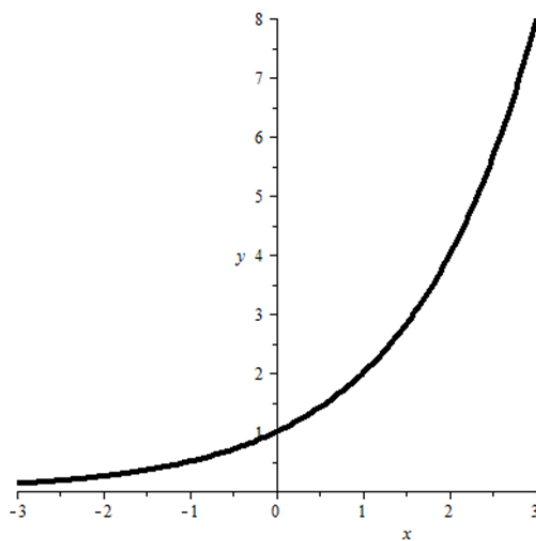
# Exponential function

## Definition

f: $D \rightarrow \mathbb{R}$	$(D \subseteq \mathbb{R})$
$x \rightarrow y = f(x) = c \cdot a^x$	$(a \in \mathbb{R}^+ \setminus \{1\}, c \in \mathbb{R} \setminus \{0\})$
$a > 1$ : exponential <b>growth</b>	
$a < 1$ : exponential <b>decay</b>	

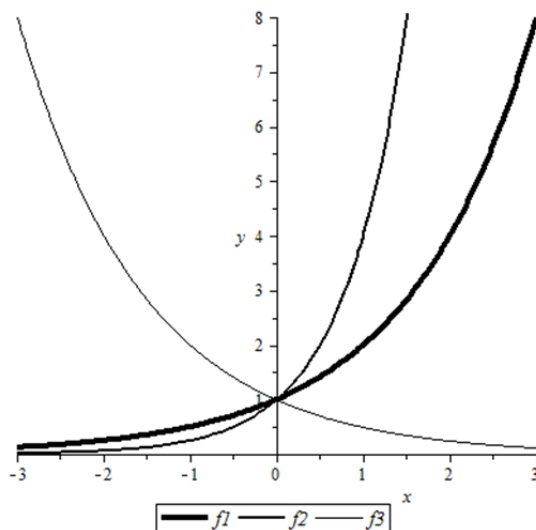
## Graph

1.  $y = f(x) = 2^x$   $(c = 1, a = 2)$



2. Parameter a

$y = f_1(x) = 2^x$	$(c = 1, a = 2)$
$y = f_2(x) = 4^x$	$(c = 1, a = 4)$
$y = f_3(x) = \left(\frac{1}{2}\right)^x$	$\left(c = 1, a = \frac{1}{2}\right)$

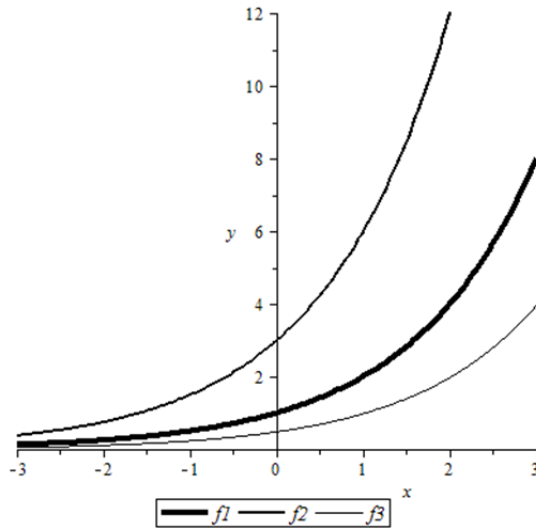


3. Parameter c

$$y = f_1(x) = 2^x \quad (a = 1, c = 1)$$

$$y = f_2(x) = 3 \cdot 2^x \quad (a = 1, c = 3)$$

$$y = f_3(x) = \frac{1}{2} \cdot 2^x \quad \left(a = 1, c = \frac{1}{2}\right)$$

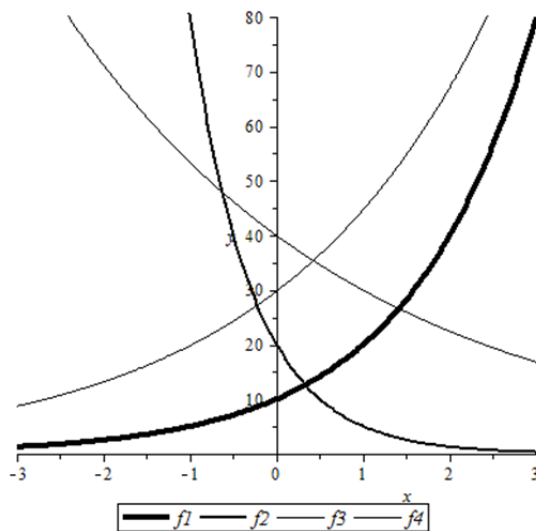


4.  $y = f_1(x) = 10 \cdot 2^x \quad (c = 10, a = 2)$

$$y = f_2(x) = 20 \cdot 0.25^x \quad (c = 20, a = 0.25)$$

$$y = f_3(x) = 40 \cdot 0.75^x \quad (c = 40, a = 0.75)$$

$$y = f_4(x) = 30 \cdot 1.5^x \quad (c = 30, a = 1.5)$$



## Examples

1. Compound interest (exponential **growth**)

$$C_n = C_0 \cdot q^n$$

$C_0$  = initial capital  
 $C_n$  = capital after n compounding periods  
n = number of compounding periods (typically: 1 compounding period = 1 year)  
q = growth factor = 1 + r (q > 1)  
r = interest rate per compounding period

Ex.:  $C_0 := 1000, r := 2\% = 0.02 \Rightarrow q = 1.02 \Rightarrow C_n = 1000 \cdot 1.02^n$

2. Consumer price index (exponential **decay**)

$$P(t) = P_0 \cdot q^t$$

$P_0$  = initial purchasing power  
P(t) = purchasing power at time t (typically: t in years)  
q = decay factor (q < 1)

Ex.:  $P_0 := 100, q := 0.97 \Rightarrow P(t) = 100 \cdot 0.97^t$