

Exercises 16 Indefinite integral Antiderivative, indefinite integral, coefficient/sum rule

Objectives

- be able to determine an antiderivative and the indefinite integral of a constant/basic power/basic exponential function.
- be able to apply the coefficient/sum rule to determine the indefinite integral of a function.
- be able to determine the cost/average cost/revenue/profit function if the marginal cost/average cost/revenue/profit function is known.

Problems

16.1 Determine the indefinite integrals below:

- a) $\int x^3 dx$
- b) $\int x^2 dx$
- c) $\int \frac{1}{x^4} dx$
- d) $\int \frac{1}{x^2} dx$
- e) $\int x^{-5} dx$
- f) $\int 4 dx$
- g) $\int (-7) dx$
- h) $\int e^x dx$

16.2 Determine the indefinite integral of the following functions f:

- a) $f(x) = x^5$
- b) $f(x) = 3x^2$
- c) $f(x) = x^3 + 2x^2 - 5$
- d) $f(x) = \frac{1}{2}x^5 - \frac{2}{3x^2}$
- e) $f(x) = \frac{1}{2}x^3 - 2x^2 + 4x - 5$
- f) $f(x) = x^{10} - \frac{1}{2}x^3 - x$

16.3 Find two antiderivatives $F_1(x)$ and $F_2(x)$ of $f(x)$ such that the stated conditions are fulfilled.

- a) $f(x) = 10x^2 + x$ $F_1(0) = 3$ $F_2(0) = -1$
- b) $f(x) = x^3 + 3x + 1$ $F_1(2) = 5$ $F_2(4) = -8$

16.4 Suppose that we know the equation of the derivative f' of a function f :

$$f'(x) = 3x^2 - 50x + 250$$

Determine the equation of the function f , if ...

- a) ... $f(0) = 500$.
- b) ... $f(10) = 2500$.

16.5 Suppose that we know the equation of the second derivative f'' of a function f :

$$f''(x) = 2x - 1$$

Find the equation of ...

- a) ... the first derivative f' such that $f'(2) = 4$.
- b) ... the function f such that $f'(2) = 4$ and $f(1) = -1$.

16.6 If the monthly marginal cost (in dollars) for a product is $C'(x) = 2x + 100$, with fixed costs amounting to \$200, find the total cost function for the month.

16.7 If the marginal cost (in dollars) for a product is $C'(x) = 4x + 2$, and the production of 10 units results in a total cost of \$300, find the total cost function.

16.8 If the marginal cost (in dollars) for a product is $C'(x) = 4x + 40$, and the total cost of producing 25 units is \$3000, what will be the cost of producing 30 units?

16.9 A firm knows that its marginal cost for a product is $C'(x) = 3x + 20$, that its marginal revenue is $R'(x) = 44 - 5x$, and that the cost of production and sale of 80 units is \$11'400.

- a) Find the profit function $P(x)$.
- b) How many units will result in a maximum profit?

Hint:

- The revenue R is zero if no unit is sold. Thus, $R(0) = \$0$.

16.10 Suppose that the marginal revenue $R'(x)$ and the derivative of the average cost $\bar{C}'(x)$ are given as follows:

$$R'(x) = 100$$
$$\bar{C}'(x) = 2 - \frac{1800}{x^2}$$

The production of 10 units results in a total cost of \$1000.

- a) Find the total cost function $C(x)$.
- b) How many units will result in a maximum profit? Find the maximum profit.

Answers

16.1 a) $\int x^3 dx = \frac{x^4}{4} + C$

b) $\int x^2 dx = \frac{x^3}{3} + C$

c) $\int \frac{1}{x^4} dx = -\frac{1}{3x^3} + C$

d) $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

e) $\int x^{-5} dx = -\frac{1}{4x^4} + C$

f) $\int 4 dx = 4x + C$

g) $\int (-7) dx = -7x + C$

h) $\int e^x dx = e^x + C$

16.2 a) $\int f(x) dx = \int x^5 dx = \frac{x^6}{6} + C$

b) $\int f(x) dx = \int 3x^2 dx = x^3 + C$

c) $\int f(x) dx = \int (x^3 + 2x^2 - 5) dx = \frac{x^4}{4} + \frac{2x^3}{3} - 5x + C$

d) $\int f(x) dx = \int \left(\frac{1}{2}x^5 - \frac{2}{3x^2}\right) dx = \frac{x^6}{12} + \frac{2}{3x} + C$

e) $\int f(x) dx = \int \left(\frac{1}{2}x^3 - 2x^2 + 4x - 5\right) dx = \frac{x^4}{8} - \frac{2x^3}{3} + 2x^2 - 5x + C$

f) $\int f(x) dx = \int \left(x^{10} - \frac{1}{2}x^3 - x\right) dx = \frac{x^{11}}{11} - \frac{x^4}{8} - \frac{x^2}{2} + C$

16.3 a) $F_1(x) = \frac{10x^3}{3} + \frac{x^2}{2} + 3$ $F_2(x) = \frac{10x^3}{3} + \frac{x^2}{2} - 1$

b) $F_1(x) = \frac{x^4}{4} + \frac{3x^2}{2} + x - 7$ $F_2(x) = \frac{x^4}{4} + \frac{3x^2}{2} + x - 100$

Hints:

- First, determine the indefinite integral of $f(x)$.
- Then, find the value of the integration constant such that the stated condition is fulfilled.

16.4 a) $f(x) = x^3 - 25x^2 + 250x + 500$

b) $f(x) = x^3 - 25x^2 + 250x + 1500$

16.5 a) $f'(x) = x^2 - x + 2$

b) $f(x) = \frac{x^3}{3} - \frac{x^2}{2} + 2x - \frac{17}{6}$

16.6 $C(x) = x^2 + 100x + 200$

Hints:

- First integrate the marginal cost function $C'(x) \Rightarrow C(x) = x^2 + 100x + C$ ($C \in \mathbb{R}$)
- Determine the integration constant C using the fact that $C(0) = \$200 \Rightarrow C = 200$

16.7 $C(x) = 2x^2 + 2x + 80$

16.8 $C(30) = \$3750$

Hint:

- First, determine the cost function $C(x) \Rightarrow C(x) = 2x^2 + 40x + 750$.

16.9 a) $P(x) = -4x^2 + 24x - 200$

Hints:

- Find the cost and revenue functions $C(x)$ and $R(x) \Rightarrow C(x) = \frac{3}{2}x^2 + 20x + 200$, $R(x) = 44x - \frac{5}{2}x^2$

- Then, determine the profit function $P(x)$.

b) $x = 3$

Hints:

- Find the relative maximum of the profit function $P(x)$.

- Check if the relative maximum is the absolute maximum.

16.10 a) $C(x) = 2x^2 - 100x + 1800$

Hints:

- First, determine the average cost function $\bar{C}(x) \Rightarrow \bar{C}(x) = 2x + \frac{1800}{x} + C_1$

- Then, determine the cost function $C(x)$.

b) $P = \$3200$ is the absolute maximum profit at $x = 50$ units.

Hints:

- First, determine the revenue function $R(x) \Rightarrow R(x) = 100x$

- Then, find the profit function $P(x) \Rightarrow P(x) = -2x^2 + 200x - 1800$

- Find the relative maximum of the profit function $P(x)$.

- Check if the relative maximum is the absolute maximum.