Exercises 5 Linear function and equations
Linear equations

Objectives

- be able to solve a linear equation.
- be able to determine the solution set of a linear equation.
- be able to solve a linear equation containing parameters.
- be able to treat applied tasks in economics by means of linear equations.

Problems

5.1 Determine the solution sets of the following equations:

a) $19x - 32 + 17x = 18x - 30 + 16x - 4$

b) $25x - 16 - 9x = 20 + 24x - 10 - 10x$

c) $105 - 72x - 53 - 69 = 55x + 43x - 23 - 170x + 6$

d) $56x - 43 - 52 - 19x = 7 - 72x - 56x + 165x - 112$

5.2 Determine the solution sets of the following equations:

a) $22(x - 11) - 5(x - 40) = 110 - (x + 53)$

b) $184 - 6(x - 24) = 214 - 3(2x - 38)$

c) $(x + 3)(x - 5) = (x - 3)^2$

d) $(x - 5)(x - 2) = (x - 4)(x - 3)$

e) $5x(x - 1) - (2x + 3)^2 - (x - 5)(x + 3) - 6 = 0$

5.3 Determine the solution sets of the following equations:

a) $\frac{x + 3}{5} = \frac{2x - 8}{3}$

b) $\frac{x + 3}{4} + \frac{1 - 3x}{7} = 0$

c) $\frac{2}{x - 1} = \frac{1}{x - 2}$

d) $\frac{x}{x - 1} = \frac{x - 1}{x - 2}$

5.4 The equations below are equations in the variable x. Furthermore, the equations contain real parameters a and b. Therefore, the solution sets of the equations depend on the values of these parameters.

Solve the equations for x, and determine the solution sets.
Take into account that the parameters a and b can be any real numbers.

a) $x(a - 3) = a$

Hint:
- You may want to divide both sides of the equation by a - 3. However, this is not allowed if a - 3 = 0, i.e. if a = 3, as dividing by 0 is not defined.

b) $(x + 1)(b - 2) = 2bx$

c) $a(1 + x) - 1 = x(2a - 1)$

d) $(a - b)x = a$

5.5 The graph of a linear function f with slope a contains the point P. Find the equation of the linear function.

a) a = -5 P(5|-3)

b) a = 2 P(3|0)

c) a = 0 P(2|3)

5.6 Alps Bikes uses the formula B(t) = - 400 t + 5000 to find the book value B(t), in Swiss franks, of a mountain bike t years after its purchase.

a) What do the numbers - 400 and 5000 signify?

b) How long will it take the mountain bike to depreciate completely?

5.7 Two items A and B depreciate linearly:

Item A original value = 200 CHF
depreciation = 16 CHF/year

Item B original value = 240 CHF
depreciation = 32 CHF/year

a) How long will it take the two items to depreciate completely?

b) Determine the point in time where both items have the same value.

Hint:
- Think of the temporal development of the values as linear functions.

5.8 Simple interest at an unknown rate is paid on an initial bank balance of 5000 CHF. The balance after five years is 5625 CHF.

a) Determine the interest rate.

b) How long would it take the balance to reach 7000 CHF?

Answers

5.1 a) $S = \{-1\}$

b) $S = \{13\}$

c) $S = \mathbb{R}$

d) $S = \{\ \}$

5.2 a) $S = \left\{\frac{11}{2}\right\}$

b) $S = \mathbb{R}$

c) $S = \{6\}$

d) $S = \{\ \}$

e) $S = \{0\}$

5.3 a) $S = \{7\}$

b) $S = \{5\}$

c) $S = \{3\}$

d) $S = \{\ \}$

5.4 a) if $a = 3$: no solution $\Rightarrow$ $S = \{\ \}$
if $a \neq 3$: $x = \frac{a}{a-3}$ $\Rightarrow$ $S = \left\{\frac{a}{a-3}\right\}$

b) if $b = -2$: no solution $\Rightarrow$ $S = \{\ \}$
if $b \neq -2$: $x = \frac{b-2}{b+2}$ $\Rightarrow$ $S = \left\{\frac{b-2}{b+2}\right\}$

c) if $a = 1$: $x \in \mathbb{R}$ $\Rightarrow$ $S = \mathbb{R}$
if $a \neq 1$: $x = 1$ $\Rightarrow$ $S = \{1\}$

d) if $a = b = 0$: $x \in \mathbb{R}$ $\Rightarrow$ $S = \mathbb{R}$
if $a = b \neq 0$: no solution $\Rightarrow$ $S = \{\ \}$
if $a \neq b$: $x = \frac{a}{a-b}$ $\Rightarrow$ $S = \left\{\frac{a}{a-b}\right\}$

5.5 a) $y = f(x) = -5x + 22$

Hints:
- The equation of a linear function is $y = f(x) = ax + b$
- If P(5|-3) is a point of the graph of the linear function, its coordinates must fulfil the equation of the linear function, i.e. $-3 = f(5) = a \cdot 5 + b$

b) $y = f(x) = 2x - 6$

c) $y = f(x) = 3$

5.6 a) The number - 400 indicates that the value of the mountain bike decreases by 400 CHF per year.
The number 5000 indicates that the original value of the mountain bike was 5000 CHF.

b) 12.5 years

Hint:
- Complete depreciation at time t means $B(t) = 0$.

5.7 a) item A: 12.5 years
item B: 7.5 years

b) t = 2.5 years (if the original value is meant to be at t = 0 years)

5.8 a) slope of the linear function: a = 125
interest rate r = 2.5%

b) 16 years