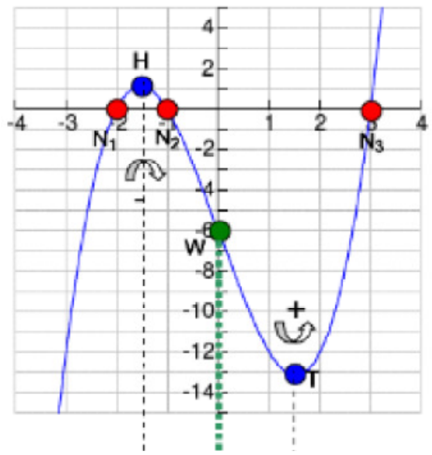
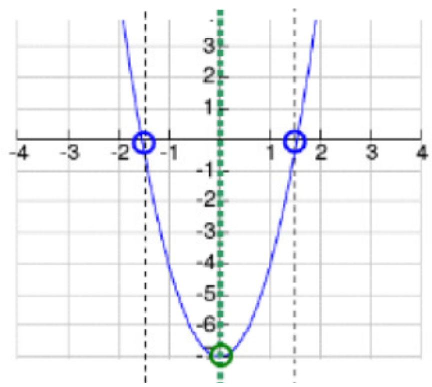
Increasing/decreasing, concavity

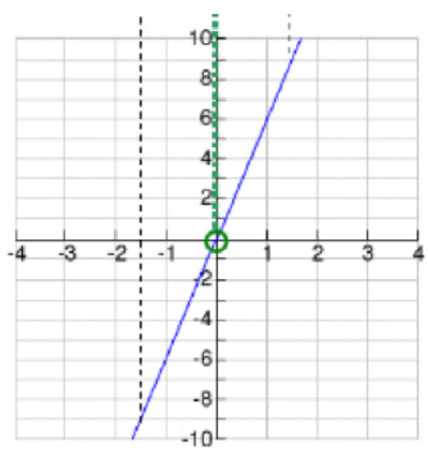
Ex.: $f(x) = x^3 - 7x - 6$



$f'(x) = 3x^2 - 7$



$f''(x) = 6x$



Increasing/decreasing

The function f is **increasing** at $x = x_0$, if the **first derivative** is **positive**, i.e. $f'(x_0) > 0$.

The function f is **decreasing** at $x = x_0$, if the **first derivative** is **negative**, i.e. $f'(x_0) < 0$.

Concavity

The graph of the function f is **concave up** at $x = x_0$, if the **second derivative** is **positive**, i.e. $f''(x_0) > 0$.

The graph of the function f is **concave down** at $x = x_0$, if the **second derivative** is **negative**, i.e. $f''(x_0) < 0$.

Relative maxima/minima

The function f has a **relative maximum** at $x = x_0$, if $f'(x_0) = 0$ and $f''(x_0) < 0$.

The function f has a **relative minimum** at $x = x_0$, if $f'(x_0) = 0$ and $f''(x_0) > 0$.

Note:
A **relative** maximum/minimum is not necessarily an **absolute** maximum/minimum.

Points of inflection

The function f has a **point of inflection** at $x = x_0$, if $f''(x_0) = 0$ and $f'''(x_0) \neq 0$.

Ex.: $f(x) = x^3 - 7x - 6$ (see page 1)

$f'(x) = 3x^2 - 7$

$f''(x) = 6x$

$f'''(x) = 6$

$f'(x) = 0$ at $x_1=\sqrt{\frac{7}{3}} = 1.52\ldots$ and $x_2= -\sqrt{\frac{7}{3}} = -1.52\ldots$

$f''(x_1) = 6 \cdot \sqrt{\frac{7}{3}} = 9.16\ldots > 0$ $\Rightarrow$ relative minimum at $x_1=\sqrt{\frac{7}{3}}$

$f''(x_2) = -6 \cdot \sqrt{\frac{7}{3}} = -9.16\ldots < 0$ $\Rightarrow$ relative maximum at $x_2= -\sqrt{\frac{7}{3}}$

$f''(x) = 0$ at $x_3 = 0$

$f'''(x_3) = 6 \neq 0$ $\Rightarrow$ point of inflection at $x_3 = 0$

Financial mathematics

Marginal cost/revenue/profit function = first derivative of the cost/revenue/profit function

Ex.: Cost function $C(x) = 120x + x^2$
$\Rightarrow$ Marginal cost function $C'(x) = 120 + 2x$

Revenue function $R(x) = 168x - 0.2x^2$
$\Rightarrow$ Marginal revenue function $R'(x) = 168 - 0.4x$

Profit function $P(x) = R(x) - C(x) = 48x - 1.2x^2$
$\Rightarrow$ Marginal profit function $P'(x) = 48 - 2.4x$

Average cost/revenue/profit function

Average cost function $\bar{C}(x) := \frac{C(x)}{x}$ where C(x) = cost function

Ex.: Cost function $C(x) = 3x^2 + 4x + 2$
$\Rightarrow$ Average cost function $\bar{C}(x) = 3x + 4 + \frac{2}{x}$

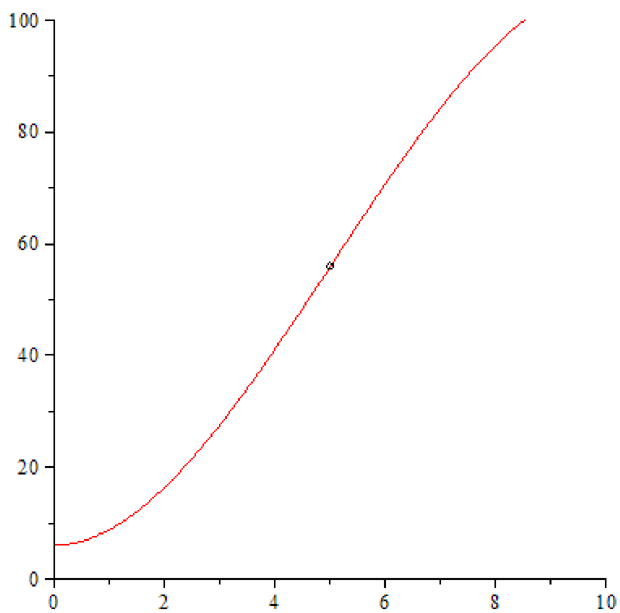
Average revenue function $\bar{R}(x) := \frac{R(x)}{x}$ where R(x) = revenue function

Average profit function $\bar{P}(x) := \frac{P(x)}{x}$ where P(x) = profit function

Point of diminishing returns

Point of diminishing returns = point of inflection on the graph

Ex.: Profit function
$P(x) = -0.2x^3 + 3x^2 + 6$



Point of diminishing returns: (5|56)