Exercises 13 Derivative
Rate of change, derivative of constant/power/exponential functions

Objectives

- be able to determine rates of change of a constant /linear function.
- be able to determine rates of change of a basic power/exponential function.
- be able to determine the derivative of a constant/linear function.
- be able to determine the derivative of a basic power/exponential function.

Problems

13.1 Determine the rate of change $f'(x_0)$ at x_0 for the following functions f:

a) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = 3$ $\quad x_0 = 2$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = c \ (c \in \mathbb{R})$ $\quad$ any $x_0 \in \mathbb{R}$

c) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = 2x - 3$ $\quad x_0 = 4$

d) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = mx + q \ (m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R})$ $\quad$ any $x_0 \in \mathbb{R}$

e) * $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = |x|$ $\quad$ any $x_0 \in \mathbb{R}$

13.2 Determine the derivatives f' of the functions f below:

a) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = c \ (c \in \mathbb{R})$

b) $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = mx + q \ (m \in \mathbb{R} \setminus \{0\}, q \in \mathbb{R})$

c) * $f: \mathbb{R} \to \mathbb{R}$, $x \to y = f(x) = |x|$

13.3 Look at the function f and its derivative f':

$f: D \to \mathbb{R}$, $x \to y = f(x) = 24\sqrt{x+1} - 2x - 60$

$f': D_1 \to \mathbb{R}$, $x \to y = f'(x) = \frac{12}{\sqrt{x+1}} - 2$

Determine the largest possible …

a) … domain D of f.

b) … domain D_1 of f'.

13.4 Determine f'(x):

a) $f(x) = 3$ b) $f(x) = 0$ c) $f(x) = -1$

d) $f(x) = x^3$ e) $f(x) = x^4$ f) $f(x) = x^5$

g) $f(x) = x^{17}$ h) $f(x) = x^{200}$ i) $f(x) = x^{100'001}$

j) $f(x) = x^{-1}$ k) $f(x) = x^{-2}$ l) $f(x) = x^{-17}$

m) $f(x) = \frac{1}{x}$ n) $f(x) = \frac{1}{x^3}$ o) $f(x) = \frac{1}{x^{99}}$

13.5 Determine f'(x):

a) $f(x) = \sqrt{x}$ b) $f(x) = \sqrt[3]{x}$ c) $f(x) = \sqrt[4]{x}$

d) $f(x) = \sqrt{x^3}$ e) $f(x) = \sqrt[5]{x^3}$ f) $f(x) = \sqrt[8]{x^{-5}}$

g) $f(x) = \sqrt{\frac{1}{x}}$ h) $f(x) = \sqrt{\frac{1}{x^3}}$ i) $f(x) = \sqrt[3]{\frac{1}{x^{17}}}$

13.6 Determine f'(x):

a) $f(x) = 3^x$ b) $f(x) = 5^x$ c) $f(x) = 18^x$

d) $f(x) = \left(\frac{2}{3}\right)^x$ e) $f(x) = \left(\frac{13}{17}\right)^x$ f) $f(x) = \left(\frac{1}{4}\right)^x$

g) $f(x) = \left(\frac{1}{e}\right)^x$ h) $f(x) = \left(\frac{3}{e}\right)^x$ i) $f(x) = \left(\frac{e}{3}\right)^x$

13.7 Determine the rate of change of the function f at the indicated values of x:

a) $f(x) = x$

i) $x = 0$ ii) $x = 1$ iii) $x = -2$

b) $f(x) = x^5$

i) $x = 0$ ii) $x = 2$ iii) $x = -\frac{2}{3}$

c) $f(x) = x^{-4}$

i) $x = -1$ ii) $x = -\frac{4}{3}$ iii) $x = \sqrt[3]{2}$

d) $f(x) = \sqrt[7]{x^5}$

i) $x = 1$ ii) $x = -2$ iii) $x = \frac{1}{2}$

e) $f(x) = \left(\frac{2}{3}\right)^x$

i) $x = 0$ ii) $x = 1$ iii) $x = -2$

13.8 * The rate of change $f'(x_0)$ of f at x_0 can be determined by looking at the secant through the points $A(x_0 \mid f(x_0))$ and $B(x_0+\Delta x \mid f(x_0+\Delta x))$ of the graph of f. The slope of this secant tends towards the slope of the tangent at $A(x_0 \mid f(x_0))$ as Δx tends towards 0.

It has been showed in class how to find $f'(x_0)$ in that way for the quadratic function $f(x) = x^2$.

Find $f'(x_0)$ for the following functions:

a) $f(x) = x^3$ b) $f(x) = \frac{1}{x^2}$

Answers

13.1 a) $f'(2) = 0$ b) $f'(x_0) = 0$

c) $f'(4) = 2$ d) $f'(x_0) = m$

e) * $f'(x_0) = \begin{cases} 1 & (x_0 > 0) \\ -1 & (x_0 < 0) \\ \text{not defined} & (x_0 = 0) \end{cases}$

13.2 a) $f': \mathbb{R} \to \mathbb{R}$
$x \to y = f'(x) = 0$

b) $f': \mathbb{R} \to \mathbb{R}$
$x \to y = f'(x) = m$

c) * $f': \mathbb{R}\backslash\{0\} \to \mathbb{R}$
$x \to y = f'(x) = \begin{cases} 1 & (x>0) \\ -1 & (x<0) \end{cases}$

13.3 a) $D = \{x \in \mathbb{R} \mid x \geq -1\}$ b) $D_1 = \{x \in \mathbb{R} \mid x > -1\}$

Hints:
- The square root of a negative number is not defined.
- Division by zero is not defined.

13.4 a) $f'(x) = 0$ b) $f'(x) = 0$ c) $f'(x) = 0$

d) $f'(x) = 3x^2$ e) $f'(x) = 4x^3$ f) $f'(x) = 5x^4$

g) $f'(x) = 17x^{16}$ h) $f'(x) = 200x^{199}$ i) $f'(x) = 100'001x^{100'000}$

j) $f'(x) = -x^{-2}$ k) $f'(x) = -2x^{-3}$ l) $f'(x) = -17x^{-18}$

m) $f'(x) = -\frac{1}{x^2}$ n) $f'(x) = -\frac{3}{x^4}$ o) $f'(x) = -\frac{99}{x^{100}}$

13.5 a) $f'(x) = \frac{1}{2\sqrt{x}}$ b) $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$ c) $f'(x) = \frac{1}{4\sqrt[4]{x^3}}$

d) $f'(x) = \frac{2}{3\sqrt{x}}$ e) $f'(x) = \frac{3}{5\sqrt[5]{x^2}}$ f) $f'(x) = -\frac{5}{8\sqrt[8]{x^{13}}}$

g) $f'(x) = -\frac{1}{2\sqrt{x^3}}$ h) $f'(x) = -\frac{3}{2\sqrt{x^5}}$ i) $f'(x) = -\frac{17}{3\sqrt[3]{x^{20}}}$

Hint:
- A root can be written as a power, see formulary ("1. Powers / Roots")

13.6 a) $f'(x) = 3^x \ln(3)$ b) $f'(x) = 5^x \ln(5)$ c) $f'(x) = 18^x \ln(18)$

d) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$ e) $f'(x) = \left(\frac{13}{17}\right)^x \ln\left(\frac{13}{17}\right)$ f) $f'(x) = -\frac{\ln(4)}{4^x}$

g) $f'(x) = -\frac{1}{e^x}$ h) $f'(x) = \left(\frac{3}{e}\right)^x (\ln(3) - 1)$ i) $f'(x) = \left(\frac{e}{3}\right)^x (1 - \ln(3))$

13.7 a) $f'(x) = 1$

i) $f'(0) = 1$ ii) $f'(1) = 1$ iii) $f'(-2) = 1$

b) $f'(x) = 5x^4$

i) $f'(0) = 0$ ii) $f'(2) = 80$ iii) $f'\left(-\frac{2}{3}\right) = \frac{80}{81}$

c) $f'(x) = -\frac{4}{x^5}$

i) $f'(-1) = 4$ ii) $f'\left(-\frac{4}{3}\right) = \frac{243}{256}$ iii) $f'\left(\sqrt[3]{2}\right) = -\frac{4}{\sqrt[3]{32}}$

d) $f'(x) = \frac{5}{7\sqrt[7]{x^2}}$

i) $f'(1) = \frac{5}{7}$ ii) $f'(-2) = \frac{5}{7\sqrt[7]{4}}$ iii) $f'(x) = \frac{5\sqrt[7]{4}}{7}$

e) $f'(x) = \left(\frac{2}{3}\right)^x \ln\left(\frac{2}{3}\right)$

i) $f'(0) = \ln\left(\frac{2}{3}\right)$ ii) $f'(1) = \frac{2}{3}\ln\left(\frac{2}{3}\right)$ iii) $f'(-2) = \frac{9}{4}\ln\left(\frac{2}{3}\right)$

13.8 * a) …

b) …